1. a) Write the Laplace equation in incremental form and
b) solve it for the new temperature

\[ \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \frac{\partial T}{\partial t} \]

2) \[ \alpha \left[ \frac{T_{+x} - 2T_{-x} + T_{-y}}{\Delta x^2} + \frac{T_{+y} - 2T + T_{-y}}{\Delta y^2} \right] = \frac{T' - T}{\Delta t} \]

b) If \( \Delta x = \Delta y \)

\[ T' = T + \frac{\alpha \Delta x}{\Delta x^2} (T_{+x} + T_{-x} + T_{+y} + T_{-y} - 4T) \]

\[ \# \# \# \text{, Then} \]

\[ T' = \frac{T_{+x} + T_{-x} + T_{+y} + T_{-y}}{4} \]
2. Sketch a spreadsheet solution to determine temperature profiles in a one-dimensional system as a function of position and time. Assume the solid of interest is 10 cm thick. Use 10 increments. The thermal diffusivity is 0.10 cm²/sec. Use the maximum permissible time step. Show:
   a) all pertinent equations
   b) boundary conditions,
   c) initial conditions, and
   d) the value of the time step.

<table>
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<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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</tbody>
</table>

\[
\text{Cell} C7 \Rightarrow \frac{B6 + D6}{2} \]

\[
\text{Cell} A7 \Rightarrow A6 + dt\]
3. (15) Given the data below, what is the largest time step allowed in the simple explicit method of solving a 1D USS HT problem.

\[ \alpha = 0.4 \text{ cm}^2/\text{sec} \]
\[ \Delta x = 0.2 \text{ cm} \]

\[ \frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \quad \Delta t = \frac{1}{2} \frac{\Delta x^2}{\alpha} \]

\[ = \frac{1}{2} \frac{0.2^2}{0.4 \text{ cm}^2/\text{sec}} \]

4. Describe the Dufort-Frankel Method of solving a 1D USS HT transfer problem.

\[ \Delta t = 0.05 \text{ sec} \]

\textbf{Not on this HG}

\textbf{2001 Spring}
5. The steady state temperature profile for the plate below is desired. There is a convection boundary condition on the left side as shown: Write enough equations to show how to solve for the temperature profile.

\[
T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}
\]

\[
- \frac{q_{conv}}{k} = A
\]

\[
-\left[-k \frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right] = h (T_{i,j} - T_{in})
\]

\[
+ k \frac{T_{i,j} - T_{in}}{\Delta x} = h (T_{i,j} - T_{in})
\]

\[
T_{1,j} = \frac{h T_{in} + k \Delta x T_{2j}}{h + k \Delta x}
\]

\[
T_5 = \frac{h T_{in} + k \Delta x T_{4j+1}}{h + k \Delta x}
\]