1. Describe the Mean Value Theorem of Derivatives in 25 words or fewer. Use a sketch.

2. The equation for a vibrating string is \( \frac{\partial^2 y}{\partial t^2} = a \frac{\partial^2 y}{\partial x^2} \).
   
   a) How many boundary conditions are needed to solve this equation?
   b) How many conditions in time are needed to solve this equation?
   c) Write an approximation of the equation in incremental form.

3. A question concerning the 1D USS HC equation with no generation.
   a) Derive the equation in rectilinear coordinates.
   b) Write the equation in cylindrical and spherical coordinates.

4. Use Taylor Series approximation to
   a) Derive a forward approximation of the first derivative.
   b) Show by formal mathematics the order of the error of the above approximation.

5. Write the appropriate entry for a cell in an Excel® worksheet at \( x=0 \) for a solid for each of the indicated boundary conditions at \( x = 0 \).

   \[ \text{Flux, if any} \quad T_+ \quad x=0 \quad x=\Delta x \]

   a) Fixed temperature
   b) Zero flux
   c) A fixed flux of value \( q \) in the direction (and sense) indicated

6. Complete the macro below that will fill cells B10 to K10 with the initial temperatures assigned the name “_Tinit” on the worksheet. (If you cannot remember how to get the value of “_Tinit” into the macro, you may assume the value of “_Tinit” is “12” for partial credit.)

   Sub SampleMacro()
   
   End Sub
7. Which of the below methods could be used to numerically solve a 1D USS HC problem for a time step of 10 seconds if $\alpha = 0.5 \text{ cm}^2/\text{sec}$ and $\Delta x = 2 \text{ cm}$?

a) Elementary Explicit ________ If “No”, why? ____________________________________

b) Saul’yev _______________ If “No”, why? ____________________________________

c) Crank-Nicolson__________ If “No”, why? ____________________________________

d) Simpson’s 1/3 Rule_______ If “No”, why? ____________________________________

e) Simplex ________________ If “No”, why? ____________________________________

8. Layout the solution to the following set of equations using the Gauss-Seidel Method:

\[
2x + y - 5z = -3 \\
3x + 3y - 4z = -1 \\
9x - 2y - z = -30
\]

9. Using the data in Table 1 answer the following questions:

a) What order polynomial do the data appear to observe?

b) Approximate $f(4.3)$ using a second order approximation. (Perform no arithmetic.)

Table 1. Difference Table for Interpolation

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>4.2000</td>
</tr>
<tr>
<td>2.0</td>
<td>5.9000</td>
</tr>
<tr>
<td>3.0</td>
<td>7.2000</td>
</tr>
<tr>
<td>4.0</td>
<td>8.1000</td>
</tr>
<tr>
<td>5.0</td>
<td>8.6000</td>
</tr>
<tr>
<td>6.0</td>
<td>8.7000</td>
</tr>
<tr>
<td>7.0</td>
<td>8.4000</td>
</tr>
<tr>
<td>8.0</td>
<td>7.7000</td>
</tr>
<tr>
<td>9.0</td>
<td>6.6000</td>
</tr>
</tbody>
</table>

10. Find the integral for $f(x)dx$ from $x = 2$ to 26 using Simpson's 1/3 Rule.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>11</th>
<th>14</th>
<th>17</th>
<th>20</th>
<th>23</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5</td>
<td>1</td>
<td>-3</td>
<td>-5</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>