ODEs

**Final Exam 1999S**

3. The rate of change of $z$ with $t$ is given below. At $t=1$, $z = 3$. Find $z$ when $t = 2$ by any order Runge-Kutta method. Use a step size of 1.

$$\frac{dz}{dt} = (1+t) - 0.05z^2$$

**Final Exam 2001F**

4. The rate of change of $y$ with $t$ is given below and that at $t = 0$, $y = 300$. Describe how to find $y$ over the range $0 < t < 20$ by Runge-Kutta 4th Order. You may use an algebraic description, MathCad, MatLab, or any RK Solver but do identify your work.

$$\frac{dy}{dt} = -10 + 0.2t - 0.01y^2$$

**Hour Exam #2 2005F**

6. The following system of ODE’s is to be solved with step size of 0.01 to $t = 3$.

$$\frac{dx}{dt} = f_x(t, x, y) = 0.2t + 2x / y^2$$

$$\frac{dy}{dt} = f_y(t, x, y) = 1 + 0.3 \ln(xy)$$

$t = 0 \quad x_0 = 2.1 \quad y_0 = 1$

Choose one: (If you do both, only the first one will be graded.)

a) Write out the 4th order Runge-Kutta equations to show how to proceed through the solution.

b) Write out what the MathCad Solution looks like.

**Solution:**

a) $h = \Delta t = 0.01$

$$x_{i+1} = x_i + \frac{1}{6}(k_{1x} + 2k_{2x} + 2k_{3x} + k_{4x})$$

$$y_{i+1} = y_i + \frac{1}{6}(k_{1y} + 2k_{2y} + 2k_{3y} + k_{4y})$$

where:

$$k_{1x} = f_x(t, x, y)\Delta t$$

$$k_{1y} = f_y(t, x, y)\Delta t$$

$$k_{2x} = f_x(t + \frac{\Delta t}{2}, x + \frac{k_{1x}}{2}, y + \frac{k_{1y}}{2})\Delta t$$

$$k_{2y} = f_y(t + \frac{\Delta t}{2}, x + \frac{k_{1x}}{2}, y + \frac{k_{1y}}{2})\Delta t$$

$$k_{3x} = f_x(t + \frac{\Delta t}{2}, x + \frac{k_{2x}}{2}, y + \frac{k_{2y}}{2})\Delta t$$

$$k_{3y} = f_y(t + \frac{\Delta t}{2}, x + \frac{k_{2x}}{2}, y + \frac{k_{2y}}{2})\Delta t$$

$$k_{4x} = f_x(t + \Delta t, x + k_{3x}, y + k_{3y})\Delta t$$

$$k_{4y} = f_y(t + \Delta t, x + k_{3x}, y + k_{3y})\Delta t$$

**Solution:**

b) *(Actual MathCad Sheet)*
The rate of change of \( v \) with \( t \) is given below and that at \( t = 0 \), \( v = 300 \) find \( v \) when \( t = 0.2 \) by Runge-Kutta 4th Order using a step size of 0.2

\[
\frac{dv}{dt} = -0.002v^2 + 0.35t
\]

**Final Exam 2005F**

6. The rate of change of \( v \) with \( t \) is given below and that at \( t = 0 \), \( v = 300 \) find \( v \) when \( t = 0.2 \) by Runge-Kutta 4th Order using a step size of 0.2

\[
\begin{align*}
\text{fs}(t, x, y) & := 0.2t + 2 \frac{x}{y^2} \\
\text{fy}(t, x, y) & := 1 + 0.3\ln(xy)
\end{align*}
\]

\[
I := \begin{pmatrix} 2.1 \\ 1 \end{pmatrix} \\
D(t, z) := \begin{pmatrix} \text{fs}(t, z_0, z_1) \\ \text{fy}(t, z_0, z_1) \end{pmatrix} \\
\text{Ans} := \text{rkfixed}(1, 0, 3, 300, D)
\]

\[
\begin{aligned}
\text{xf} & := \text{Ans} \begin{pmatrix} 300, 1 \\ 300, 2 \end{pmatrix} \\
\text{yf} & := \text{Ans} \begin{pmatrix} 300, 1 \\ 300, 2 \end{pmatrix}
\end{aligned}
\]

\[
\begin{aligned}
\text{xf} &= 7.198 \\
\text{yf} &= 6.507
\end{aligned}
\]