## South Dakota School Of Mines & Technology
## Mathematical Sciences Department

Math 374 | Makeup HQ 1 | Oct 15, 1999

1. a) Write the LaPlace equation in incremental form and
   b) solve it for the new temperature.

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$$

a)

$$\alpha\left[\frac{T_{+x}-2T-T_{-x}}{\Delta x^2}+\frac{T_{+y}-2T+T_{-y}}{\Delta y^2}\right]=\frac{T'-T}{\Delta t}$$

b)

If $\Delta x=\Delta y$

$$T'=T+\frac{\alpha\Delta t}{\Delta x^2}\left(T_{+x}+T_{-x}+T_{+y}+T_{-y}-4T\right)$$

If $\frac{1}{4}$, Then

$$T'=\frac{T_{+x}+T_{-x}+T_{+y}+T_{-y}}{4}$$

2. Sketch a spreadsheet solution to determine temperature profiles in a one-dimensional system as a function of position and time. Assume the solid of interest is 10 cm thick. Use 10 increments. The thermal diffusivity is 0.10 $cm^2$/sec. Use the maximum permissible time step. Show
   a) all pertinent equations
   b) boundary conditions,
   c) Initial conditions, and
   d) the value of the time step.

Cell C5 → = B5 + ΔX

| | A | B | C | D | E … |
|---|---|---|---|---|---|
| 1 | DX | 1 | | | |
| 2 | a | 0.1 | | | |
| 3 | dt | $=\frac{1}{2}\frac{\Delta x^2}{a}$ | | | |
| 4 | | x | | | |
| 5 | time | 0 | =B5+ΔX | =C5+DX → fill | → → |
| 6 | 0 | BC#1 | IC → fill → | | BC#2 |
| 7 | =A6+dt | ↓ | =B6/2+D6/2 → fill | → | ↓ |
| 8 | ↓ fill ↓ | fill | ↓ fill ↓ | Fill | fill ↓ |

Cell C7 → $=\frac{B6}{2}+\frac{D6}{2}$

Cell A7 → = A6 + dt

3. (15) Given the data below, what is the largest time step allowed in the simple explicit method of solving a 1D USS HT problem.

alpha = 0.4 cm$^2$/sec
$\Delta x$ = 0.2 cm

$$\frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \qquad \Delta t = \frac{1}{2}\frac{\Delta x^2}{\alpha}$$

$$= \frac{1}{2}\frac{0.2^2\ \text{cm}^2}{0.4\ \text{cm}^2/\text{sec}}$$

4. Describe the Dufort-Frankel Method of solving a 1D USS HT transfer problem.

$\Delta t = 0.05\text{sec}$

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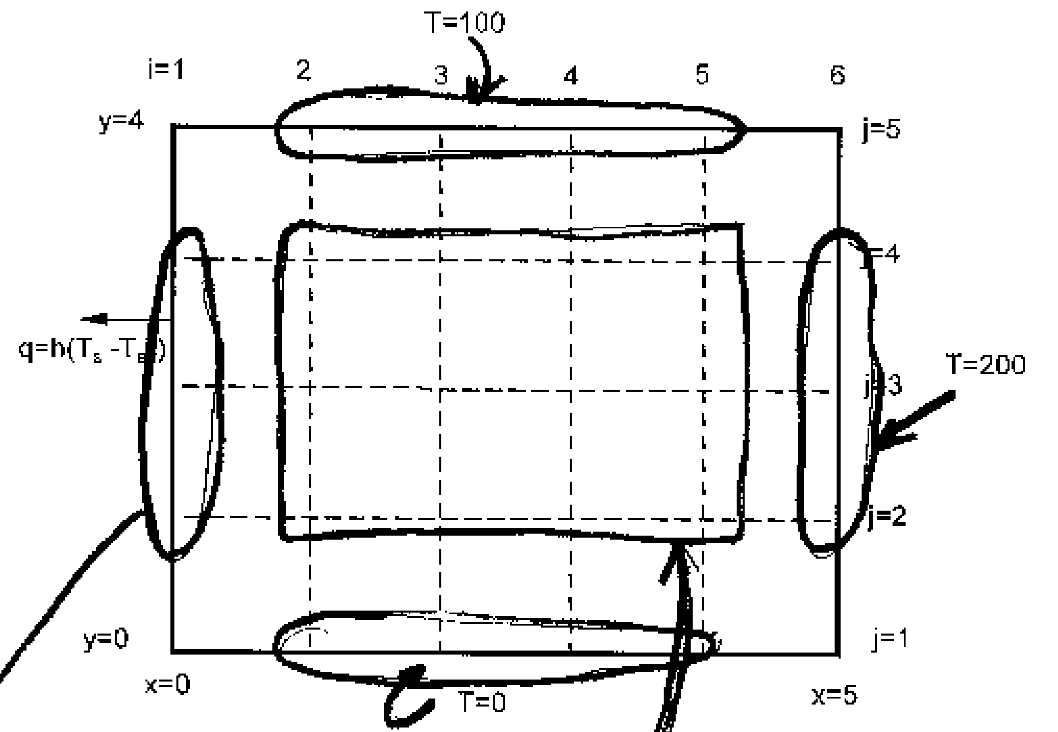
5. The steady state temperature profile for the plate below is desired. There is a convection boundary condition on the left side as shown. Write enough equations to show how to solve for the temperature profile.




$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

$q_{cond} = q_{conv}$ $\quad$ $i = 1$

$$(-1)\left(-k\frac{T_{i+1,j}-T_{i,j}}{\Delta x}\right) = h(T_{i,j}-T_{\infty})$$

$$+\frac{k(T_{2,j}-T_{1,j})}{\Delta x} = h(T_{1,j}-T_{\infty})$$

$$T_{1,j} = \frac{hT_{\infty} + k/\Delta x \, T_{2,j}}{h + k/\Delta x}$$

OR

$$T_5 = \frac{hT_{\infty} + \frac{k}{\Delta x} T_{5+1}}{h + k/\Delta x}$$