

**SOUTH DAKOTA SCHOOL OF MINES & TECHNOLOGY**  
**MATHEMATICAL SCIENCES DEPARTMENT**

Math 374

Final Exam

May 7, 1999

Do not waste time on repetitive computations until each problem you can work is set up and its solution method clearly demonstrated.

1. Use Newton's method to find a root of the following equation. Start at  $x=2$ .

$$x^3 - 70 = 0$$

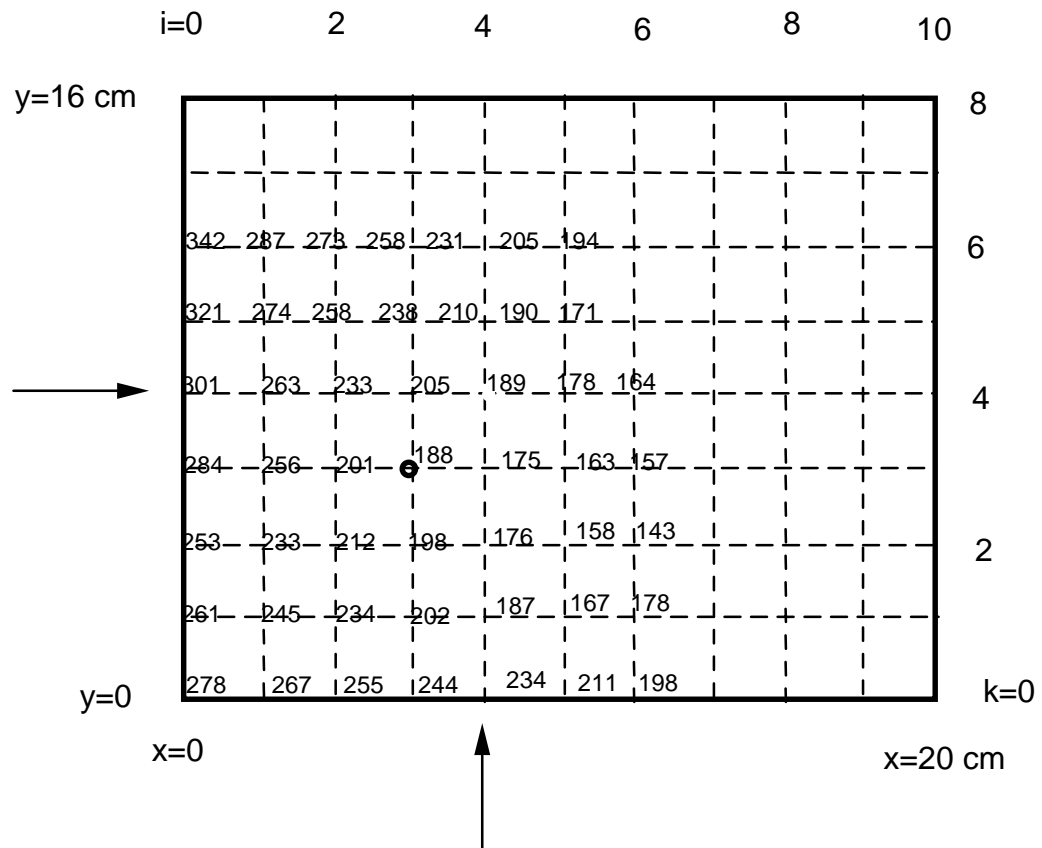
2. Find the value of  $\int_0^{0.6} f(x)dx$  by any **two** methods. Clearly identify the methods by name.

x	f(x)
0.0	2.00000
0.1	1.91984
0.2	1.67808
0.3	1.27352
0.4	0.70976
0.5	0.00000
0.6	-0.82816
0.7	-1.72792
0.8	-2.62848
0.9	-3.43024
1.0	-4.00000

3. The rate of change of  $z$  with  $t$  is given below. At  $t=1$ ,  $z = 3$ . Find  $z$  when  $t= 2$  by any order Runge-Kutta method. Use a step size of 1.

$$\frac{dz}{dt} = (1+t) - 0.05z^2$$

4. Below is a temperature profile at time = 0. Find the temperature at  $i=3$  and  $j=3$  at time = 20 seconds. Assume the largest time step consistent with a stable solution. The value of  $\alpha$  is  $0.10 \text{ cm}^2/\text{sec}$ . All dimensions are in cm. and all temperatures are in  $^{\circ}\text{C}$ . State any assumptions made.



NOTE:  $\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$

5. Write the **forward and central** derivative in incremental form for  $\frac{\partial T}{\partial t}$ .

6. Describe the Saul'yev Method of solving a 1D USS HT problem. Label the sketch for reference.

7. Solve the following set of linear equations using Gauss-Seidel

$$\begin{aligned} -x+2y+4z &= 10 \\ x+3y+2z &= 11 \\ 2x+y &= 5 \end{aligned}$$