South Dakota School of Mines and Technology
Department of Computer and Mathematical Sciences

Math 373 | Final Exam | Dec 16, 2002

Refer to the following 3 hour exams. They are either the same or approximately the same exams taken during the semester. Work any or all of the problems but scoring the exam will be perform by seeing the IMPROVEMENT on each exam material compared to your previous grade. Therefore, do not expect a higher grade by simply working a little of each exam. Your grades are available from each exam if you needed them.

Math 373 | HQ 1 | Oct 8, 2002

Turn in ONLY the printed sheets. Enter solutions in space provided ONLY.
There may be MORE data provided than needed to solve a problem.
No caluculators, notes, books, reference materials

1. a) Write the 2D USS Heat Equation in incremental form.

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$$

   b) Solve it for the new temperature at any time step.
   c) Show the solution for the maximum time step.

2. Mark the location of ξ according to the Mean Value Theorem of Derivatives

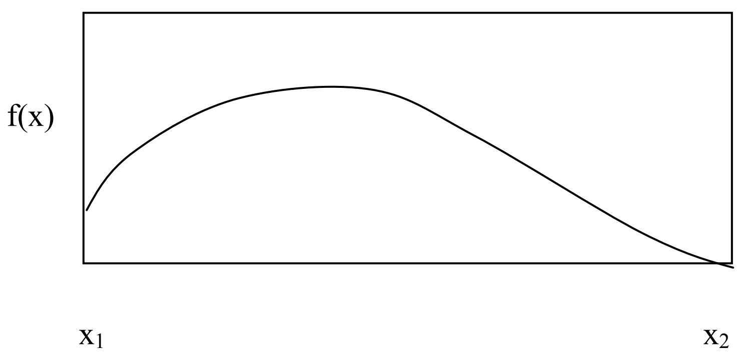

Figure 1. f(x) vs. x

3 For $f(x) = 2x^3 + 3x^2 - 5$
   a) Write the first order Taylor Series approximation in terms of x and h for the above function.
   b) What is the value of ξ that makes the first order approximation exact when x = 1 and h=0.5?

4. Derive the 1D USS HT equation in rectilinear coordinates. Include a generation term, S, per unit volume. Show your work in detail.

5. Solve the equation $\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$ for the steady state temperature as needed in the spreadsheet solution

6. Given the data below, what is the largest time step allowed in the method of solving a 1D USS HT problem by the methods covered so far in class if
   a) $\alpha = 0.5\ cm^2/sec$ and $\Delta x = 0.2$ cm?
   b) k = 1.0 J/(cm*K*sec),
      Cp = 0.5 J/(g*K), and
      $\rho = 8\ g/cm^3$?

7. Complete the macro below that will find the monthly payment given the number of payments, n, the amount borrowed, P, and the interest per period, i.

The $Payment = \mathrm{P}\dfrac{(1+i)^n - 1}{\mathrm{i}}$

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Function Payment(P, i, n)



End Function
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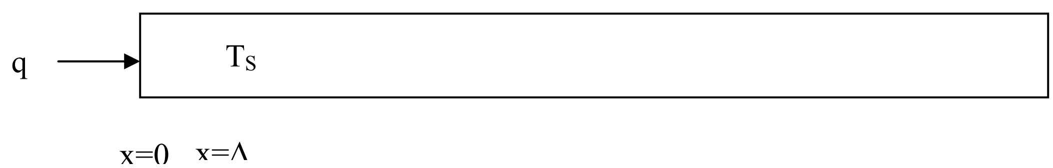
Math 373 HQ 2 Oct 30, 2002

1. A fixed heat flux of 10 Joules/(sec*cm^2) is being added into the solid steel bar below at x=0.



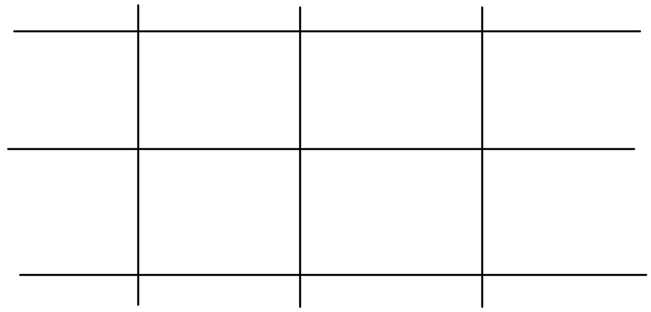
   a) Will the temperature at $x = \Delta x$ be lower, same, or higher than the temperature at x = 0?
   b) Write the equation that describes the boundary temperature's relationship to the temperature at $x = \Delta x$. Use the notation in the sketch above.

2. Which of the following methods are explicit? Clearly cross out the ones that are not, if any.
   a) ADI
   b) Saul'yev
   c) Dufort-Frankel
   d) Crank-Nicholson

3. a) How much energy can be stored at a surface with a boundary condition?
   b) Can the temperature change at a surface where a boundary conditions applies?

4. What happened to the "a" term in the first equation and the "c" term in the last equation of the tridiagonal matrix obtained in the implicit methods for determining temperature profiles in solids?

5. Describe the Dufort-Frankel Method. Be specific: equations, labeled sketch, values in equations, etc. Discuss skew, if any, in the approximations.



6. Given the Tridiagonal Matrix Algorithm below, find three errors in the following macro for the algorithm: Sub Tridiag(n, d, Tnew) **Every wrong answer will void one correct answer so be careful.**

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 ' n = number of unknown T's
' d = RHS matrix

Dim a, b, c, d(10), Tnew(10), T(10)
        .
        .
Sub Tridiag(n, d, Tnew)
Dim beta(100), gamma(100)
beta(1) = b
gamma(1) = b(1) / beta(1)
For i = 2 To n-1
beta(i) = b - a * c / beta(i)
gamma(i) = d(i) – (a * gamma(i - 1)) / beta(i)
Next I
Tnew(n) = gamma(n)
For i = n - 1 To 1 Step –1
Tnew(i) = gamma(i) - c * T(i + 1) / beta(i)
Next n
End Function
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<u>Clearly write the correct code below</u>

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Tridiagonal Matrix Algorithm

Crank-Nicholson has all the same a's, b's, and c's and are not subscripted. The general Tridagonal Algorithm allows unique values of a, b, and c in each equation and are, therefore, subscripted.

$$T_N = \gamma_N$$

$$T_i = \gamma_i - \frac{c_i T_{i+1}}{\beta_i} \qquad i = N-1,\ N-2,\ ....1$$

Where the β's and γ's are determined from the recurrision formulas

$\beta_1 = b_1$, $\qquad \gamma_1 = d_1 / \beta_1$,

$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}} \qquad i = 2, 3, ..., N$$

$$\gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \qquad i = 2, 3, ..., N.$$

Math 373 HQ 3 Dec 4, 2002

1. Using the data in Table 1 answer the following questions:
   a) What order polynomial do the data appear to observe?
   b) Approximate f(2.33) using a third order approximation. (Perform no arithmetic.)

Table 1. Difference Table for Interpolation

x	f(x)					
2.0	1.4000					
		0.4492				
2.1	1.8492		0.1518			
		0.6010		0.0206		
2.2	2.4502		0.1724		0.0010	
		0.7734		0.0216		
2.3	3.2236		0.1940		0.0010	
		0.9674		0.0226		
2.4	4.1910		0.2166		0.0010	
		1.1840		0.0235		
2.5	5.3750		0.2401		0.0010	
		1.4240		0.0245		
2.6	6.7990		0.2646		0.0010	
		1.6886		0.0254		
2.7	8.4876		0.2900			
		1.9786				
2.8	10.4662					

2. Show how to use Guassian Quadrature to determine the value of the following integrals. Be specific.
   a) $\int_{-1}^{1} (2 - 3x^2 + 9x^4) dx$
   b) $\int_{2}^{6} (2 + 3\ln(x) - x^2) dx$

3. Find the integral for f(x)dx from x = 0 to 1.6 using Simpson's 1/3 Rule.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
f(x)	3	1	-2	-5	2	7	9	10	9

4. The rate of change of y with t is given below and that at t = 0, y = 300. Describe how to find y over the range $0<t<20$ by Runge-Kutta 4th Order. You may use an algebraic description, MathCad, MatLab, or any RK Solver but do identify your work.

$$\frac{dy}{dt} = -10 + 0.2t - .01y^2$$

5. Short Answer:
   a) What is the purpose of Data Adjustment?
   b) What is the mathematical basis of the method?

6. Below are several LP tableaus in various states of completion. Describe the next step for each.

a)

x	y	z	S1	S2	S3		F	RHS
5	3	2	1	0	0		0	1000
2	2	1	0	1	0		0	200
0	5	6	0	0	1		0	100
-60	-50	-2	0	0	0		1	0

b)

x	y	z	S1	S2	S3	A	F	RHS
5	4	2	1	0	0	0	0	1000
2	10	1	0	1	0	0	0	200
0	2	6	0	0	-1	1	0	100
-40	-50	-2	0	0	0	M	1	0

c)

x	y	z	S1	S2	S3	A	F	RHS
0	3	0	1	4	3	2	0	280
0	2	1	0	1	0	-5	0	52
1	5	0	0	0	1	3	0	21
0	0	0	0	35	5	14	1	4560

d) Circle the pivot in any of the above tableaus that are ready to use a pivot.

6. Layout the solution to the following set of equations using the Gauss-Seidel Method:

$$2x + y^2 - z^2 = -3$$

$$3x^3 + 2y - 4z = -1$$

$$x^2 - 7y^2 - z = -30$$