Intermediate PDQs

Hour Exam Oct 15, 1999

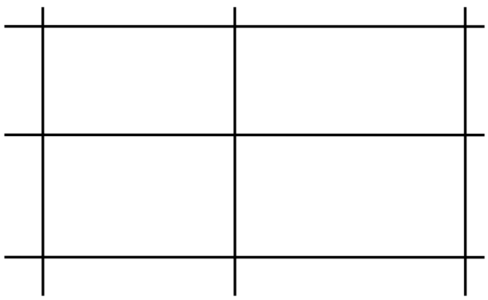
4. Describe the Dufort-Frankel Method of solving a 1D USS HT transfer problem.

Final Exam 1999S

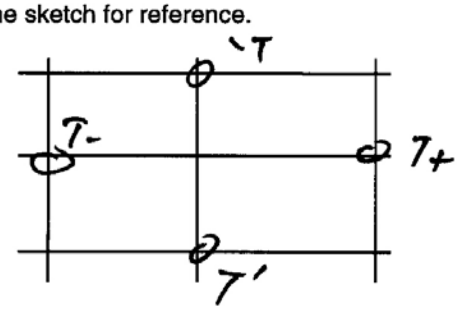
6. Describe the Saul'yev Method of solving a 1D USS HT problem. Label the sketch for reference.

Hour Exam 1998F

6. Describe the DuFort-Frankel Method of solving a 1D USS HT problem. Label the sketch for reference.



6. (15) Describe the DuFort-Frankel Method of solving a 1D USS HT problem. Label the sketch for reference.

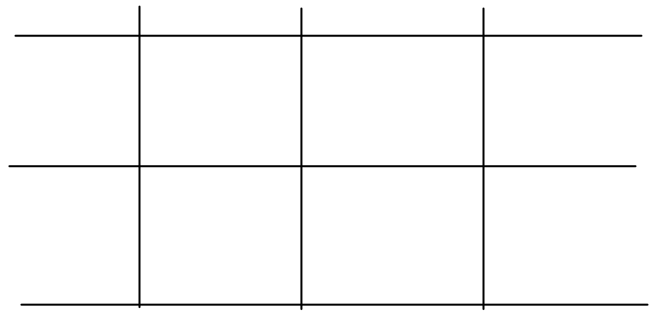


$$\alpha \frac{\frac{T_+ - T'}{\Delta x} - \frac{`T - T_-}{\Delta x}}{\Delta x} = \frac{T' - `T}{2\Delta t}$$ solve for T'

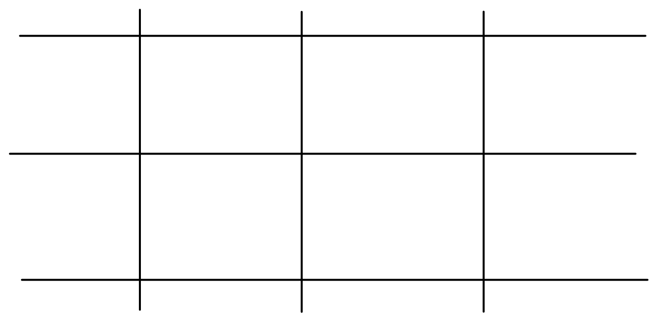
Hour Exam 2002S

2. Describe the following "fast" methods for solving a 1D USS HT problem using

   a) DuFort-Frankel Method. Use the grid and label.

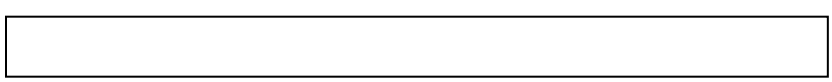


   b) Saul'yev



Final 2001F

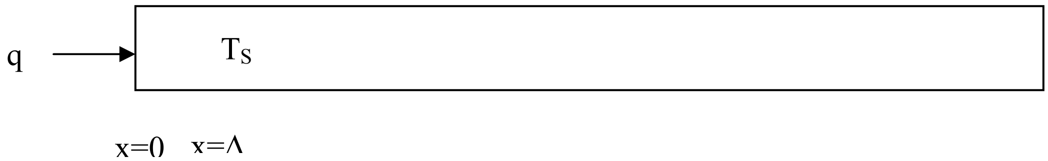
9. The solid bar below is conducting heat along its horizontal axis while heat is being lost by convection from both ends. Derive the boundary equations showing the temperature at the rod ends as a function of external fluid temperature T_a and the temperatures $T_{\Delta x}$ and $T_{L-\Delta x}$ each one increment in from each end. (Assume there is no heat loss from the surface perpendicular to the horizontal axis of the bar.) Note: $q_{conv} = h(T-T_a)$ $q_{cond} = -k(dT/dx)$



x

Final 2002F

1. A fixed heat flux of 10 Joules/(sec*cm^2) is being added into the solid steel bar below at x=0.



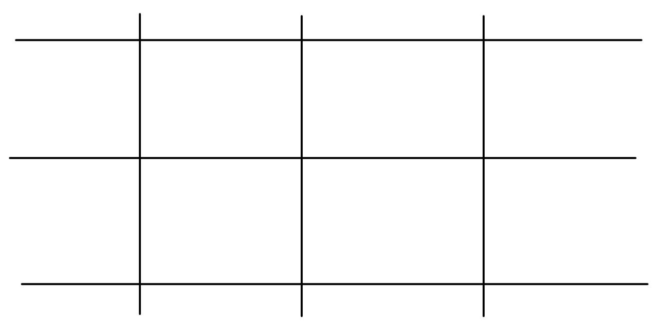
   a) Will the temperature at x = Δx be lower, same, or higher than the temperature at x = 0?
   b) Write the equation that describes the boundary temperature's relationship to the temperature at x = Δx. Use the notation in the sketch above.

1. Which of the following methods are explicit? Clearly cross out the ones that are not, if any.
   a) ADI
   b) Saul'yev
   c) Dufort-Frankel
   d) Crank-Nicholson

2. a) How much energy can be stored at a surface with a boundary condition?
   b) Can the temperature change at a surface where a boundary conditions applies?

3. What happened to the "a" term in the first equation and the "c" term in the last equation of the tridiagonal matrix obtained in the implicit methods for determining temperature profiles in solids?

5. Describe the Dufort-Frankel Method. Be specific: equations, labeled sketch, values in equations, etc. Discuss skew, if any, in the approximations.



6. Given the Tridiagonal Matrix Algorithm below, find three errors in the following macro for the algorithm: Sub Tridiag(n, d, Tnew) **Every wrong answer will void one correct answer so be careful.**

' n = number of unknown T's
' d = RHS matrix

Dim a, b, c, d(10), Tnew(10), T(10)

.

.   **<u>Clearly write the correct code below</u>**

```
Sub Tridiag(n, d, Tnew)                              ________________________
Dim beta(100), gamma(100)                            ________________________
beta(1) = b                                          ________________________
gamma(1) = b(1) / beta(1)                            ________________________
For i = 2 To n-1                                     ________________________
beta(i) = b - a * c / beta(i)                        ________________________
gamma(i) = d(i) – (a * gamma(i - 1)) / beta(i)       ________________________
Next I                                               ________________________
Tnew(n) = gamma(n)                                   ________________________
For i = n - 1 To 1 Step –1                           ________________________
Tnew(i) = gamma(i) - c * T(i + 1) / beta(i)          ________________________
Next n                                               ________________________
End Function                                         ________________________
```

Tridiagonal Matrix Algorithm
Crank-Nicholson has all the same a's, b's, and c's and are not subscripted. The general Tridagonal Algorithm allows unique values of a, b, and c in each equation and are, therefore, subscripted.

$$T_N = \gamma_N$$

$$T_i = \gamma_i - \frac{c_i T_{i+1}}{\beta_i} \qquad i = N-1,\ N-2,\ ....1$$

Where the β's and γ's are determined from the recurrision formulas

$$\beta_1 = b_1, \qquad \gamma_1 = d_1 / \beta_1,$$

$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}} \qquad i = 2, 3,\ ......,\ N$$

$$\gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \qquad i = 2, 3,\ ......,\ N.$$

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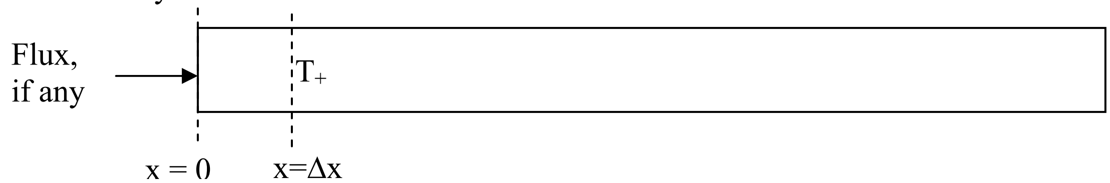
5. Write the appropriate entry for a cell in an Excel® worksheet at x=0 for a solid for each of the indicated boundary conditions at x = 0.



c) Fixed temperature
d) Zero flux
e) A fixed flux of value q in the direction (and sense) indicated

7. Which of the below methods could be used to numerically solve a 1D USS HC problem for a time step of 10 seconds if $\alpha = 0.5$ cm^2/sec and $\Delta x = 2$ cm?

a) Elementary Explicit ______ If "No", why? ______________________________
b) Saul'yev ______________ If "No", why? ______________________________
c) Crank-Nicolson__________ If "No", why? ______________________________
d) Simpson's 1/3 Rule_______ If "No", why? ______________________________
e) Simplex _______________ If "No", why? ______________________________

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2. Which of the below methods could be used to numerically solve a 1D USS HC problem for a time step of 10 seconds if $\alpha = 0.5$ cm^2/sec and $\Delta x = 2$ cm?

a) Elementary Explicit _No_____ If "No", why? Max $\Delta t = 0.5 * \Delta x^2 / \alpha = 4$ sec_____________

b) Saul'yev _Yes_____________ If "No", why? ______________________________

c) Crank-Nicolson _Yes________ If "No", why? ______________________________

d) DuFort-Frankel _Yes________ If "No", why? ______________________________

3. Complete the general Tridiagonal Matrix Algorithm macro below that will be sent N and the arrays a, b, c, and d. It will return the solved values of T. **There may be more lines than needed. Correct anything that is wrong.**

Solution**:** Underlined Text

Sub TriDiag(N, a, b, c, d, T)

Dim a(100), b(100), c(100), d(100), T(100), Beta(100), Gamma(100)

Beta(1)=b(1)

Gamma(1)=d(1)/Beta(1)

For k= 1 to N

Beta(k) = b(k) -a(k)c(k-1)/Beta(k-1)

Gamma(k) = (d(k) -a(k)Gamma(k-1)/Beta(k)

Next N

T(N) = Gamma(N)

For k= N-1 to 1 step -1

T(k) = Gamma(k)-c(k)*T(k+1)/Beta(k)

Next N

End Sub

Tridiagonal Matrix Algorithm

$$T_N = \gamma_N$$

$$T_i = \gamma_i - \frac{c_i T_{i+1}}{\beta_i} \qquad i = N-1,\ N-2,\ ....1$$

Where the β's and γ's are determined from the recurrision formulas

$$\beta_1 = b_1, \qquad \gamma_1 = d_1 / \beta_1,$$

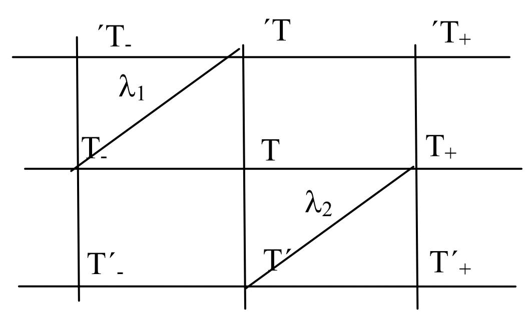
$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}} \qquad i = 2, 3, ...,\ N$$

$$\gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \qquad i = 2, 3, ...,\ N.$$

Hour Exam #2 2005F

5. Describe the following "fast" methods for solving a 1D USS HT problem using
   b) DuFort-Frankel Method. Use the grid and label everything needed in the equations written.

   Solution:



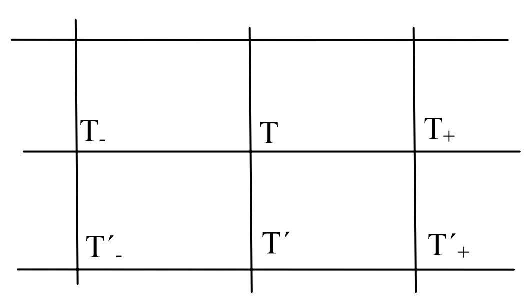
$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\delta T}{\delta t}$$

$$\alpha \frac{\lambda_2 - \lambda_1}{\Delta x} = \frac{T' - T}{2\Delta t}$$

where

$$\lambda_2 = (T_+ - T')/\Delta x \qquad \lambda_1 = (T - T_-)/\Delta x$$

Solve for T' (explicit)



b) Saul'yev

Solution:

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\delta T}{\delta t}$$

$$\alpha \frac{\lambda_2 - \lambda_1}{\Delta x} = \frac{T' - T}{2\Delta t}$$

$where:$

$$left-to-right: \lambda_2 = (T_+ - T)/\Delta x \qquad \lambda_1 = (T' - T'_-)/\Delta x$$

$$right-to-left: \lambda_2 = (T'_+ - T')/\Delta x \qquad \lambda_1 = (T - T_-)/\Delta x$$

Solve for T' (explicit)

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10. A thick, vertical plate is conducting heat through its thickness while heat is being lost by convection from both sides. Derive the boundary equation showing the temperature at the right side surface as a function of external fluid temperature T_a and the surface temperatures T_S and the temperature one increment in from the surface T_{SM}. (Assume there is no heat loss in other directions.)

Note: $q_{conv} = h(T-T_a)$
$q_{cond} = -k(dT/dx)$