Argon Sparging to Remove O as CO from Molten Steel of mass M

A mole balance on O in the steel melt gives

In – Out = Accumulation

Out= - Accumulation

$$dn_{CO} = -\frac{M}{16}\frac{d\left(wt\%O\right)}{100\%} = -\frac{Md\left(wt\%O\right)}{1600\%} \tag{1}$$

The relationship between the moles of CO in an incremental volume of argon is determined as follows:

$$dn_{CO} = d\left[\frac{P_{CO}V_T}{RT}\right]$$

$$P_{Ar}V_T = P_T V_{Ar}$$

$$V_T = \frac{P_T}{P_{Ar}}V_{Ar} = \frac{P_T}{P_T - P_{CO}}V_{Ar}$$

$$dn_{CO} = d\left[\frac{P_{CO}\left(\frac{P_T}{P_T - P_{CO}}\right)}{RT}V_{Ar}\right]$$

$$dn_{CO} = P_T\left(\frac{P_{CO}}{P_T - P_{CO}}\right)\frac{dV_{Ar}}{RT} \tag{2}$$

Substituting Eq [2] into Eq [1] and integrating gives

$$dV_{Ar} = -\frac{RTM}{1600P_T}\left[\frac{P_T}{P_{CO}} - 1\right]d\left(wt\%O\right)$$

$$= -\frac{RTM}{1600P_T}\left[\frac{P_T}{K\left(wt\%O\right)\left(wt\%C\right)} - 1\right]d\left(wt\%O\right)$$

$$V_{Ar} = \frac{RTM}{1600P_T}\left[\left(wt\%O_f - wt\%O_i\right) + \frac{P_T}{K\left(wt\%C\right)}\ln\left(\frac{wt\%O_i}{wt\%O_f}\right)\right] \tag{3}$$