South Dakota School of Mines and Technology
Department of Materials and Metallurgical Engineering

OPEN BOOK

MET 422 HQ 1 Oct 23, 2008

1. How much drag force would be exerted on a fishing line pulling a 2-cm spherical shape through water at 5 m/s?

2. How much horsepower (1 HP = 746 Watts) could be generated by capturing the energy from a water-filled leach pond 300 meters above a recovery facility if the 5-cm diameter smooth pipe between the pond and facility is 400 meters long and has three sweep elbows? The flow rate is 10 Kg/s.

3. What superficial velocity of 400 K air would be required to fluidize popcorn to a void fraction of 0.6? Assume its density is 1.2 g/cm^3 and is spherical with a diameter of 4 mm.

4. Start with the ***General Momentum Equation in Cylindrical Coordinates*** and reduce the equation to the differential equation for flow of an incompressible fluid (i.e. – constant density) in a cylindrical tube. Assume there is no flow in the radial or angular directions. Use velocity rather than momentum terms.

The Equations of Change in Curvilinear Coordinates **85**

TABLE 3.4–3

THE EQUATION OF MOTION IN CYLINDRICAL COORDINATES (r, θ, z)

In terms of $\boldsymbol{\tau}$:

r-component[a]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r \quad (A)$$

θ-component[b]

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta \quad (B)$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \quad (C)$$

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

r-component[a]

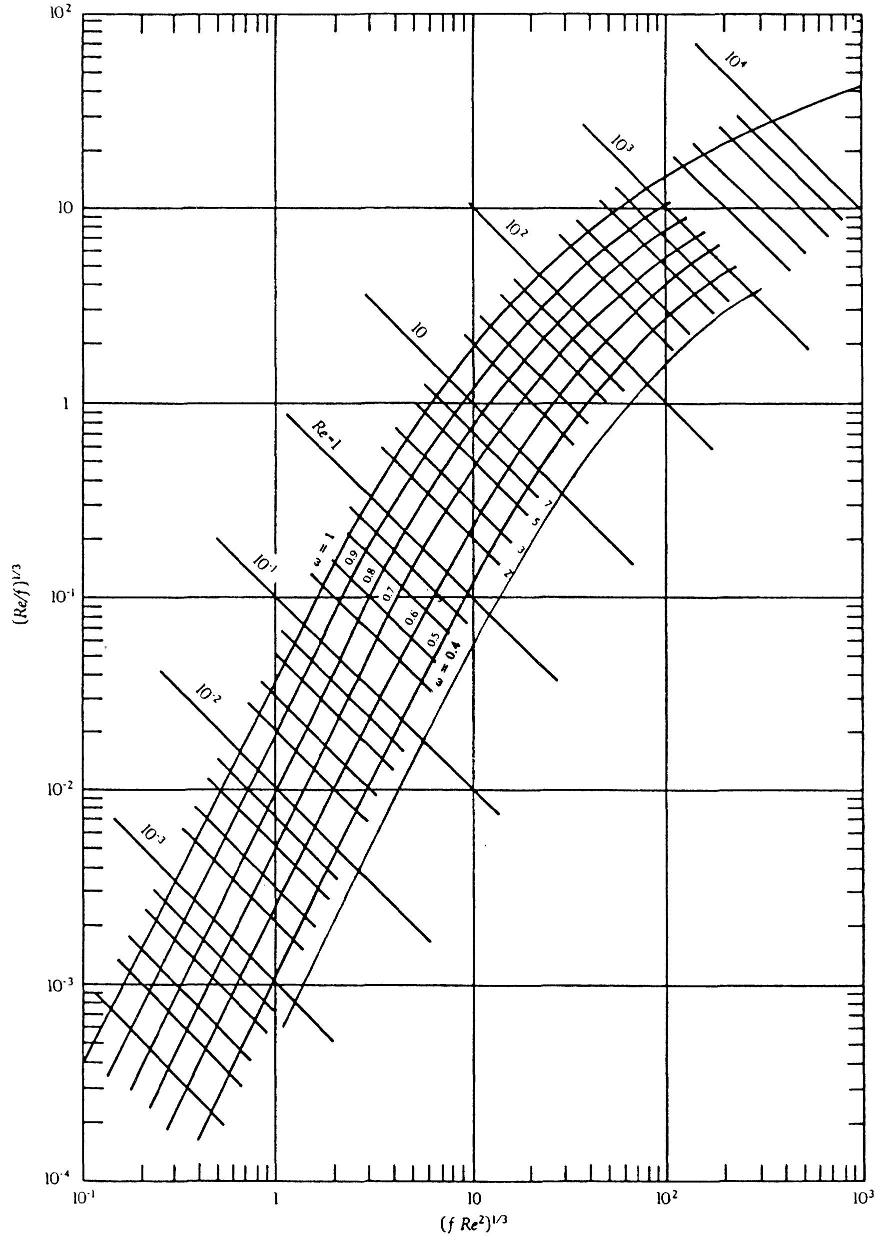
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta{}^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \quad (D)$$

θ-component[b]

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \quad (E)$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad (F)$$



$$\left[\frac{\mathrm{Re}}{f}\right]^{1/3} = \frac{V_0}{\left[\dfrac{4g\eta_f(\rho_p - \rho_f)}{3\rho_f^2}\right]^{1/3}}.$$

$$\left(f\mathrm{Re}^2\right)^{1/3} = \frac{D_p}{\left[\dfrac{3\eta_f^2}{4g\rho_f(\rho_p - \rho_f)}\right]^{1/3}}.$$