Packed Bed Solution for compressible fluids (gases)

Assume

Flow is top down from x=0 to x=L

M = mass flow rate, given as 95 kg/s

Po = inlet pressure at x=0 is $1.4\text{x}10^5$ Pa

P_L = outlet pressure at x=L, find

ρ = density of incoming gas, given as 0.5 kg/m^3

T = constant gas temperature in K, given as 800 K

A = Unrestricted cross-sectional bed area, given as pi*3^2 m^2

R = Gas Constant 0.08205/101325 Pa*m^3/(kgmol*K)

MW = Molecular weight in kg/kg mole = $\rho_o RT / P_o$ =23.754 kg/kgmole

Solution

From equation 3.49

$$\frac{\Delta P}{L} = \frac{150\eta V_o (1-\varpi)^2}{D_p^2 \omega^3} + \frac{1.75\rho V_o^2 (1-\varpi)}{D_p \omega^3}$$

note: no ρ in first term

where

$$\rho = \frac{n*MW}{V} = \frac{P*MW}{RT}$$

$$V_o = \frac{M}{A*\rho} = \frac{MRT}{A*P*MW}$$

Also,

$$P^{'} = P + \rho g (L - x)$$

$$\frac{\Delta P}{L} = -\frac{dP^{'}}{dx} = -\left(\frac{dP}{dx} - \rho g \right)$$

so,

$$-\left(\frac{dP}{dx}-\rho g\right)=\frac{150\eta V_o(1-\varpi)^2}{D_p^2\omega^3}+\frac{1.75\rho V_o^2(1-\varpi)}{D_p\omega^3}$$

Substituting in for ρ and V_o

$$\frac{dP}{dx}=\frac{A}{P}+BP$$

where

$$A=\left[\frac{150\eta(1-\varpi)^2 RTM}{D_p^2\omega^3 A*MW}+\frac{1.75(1-\varpi)M^2RT}{D_p\omega^3A^2*MW}\right]$$ (for Prob 3.10 =1.515x10^10 kg²*m⁻³ s-⁴)

$$B=\frac{MW}{RT}g$$ (for prob 3.10 = -4.839x10⁻⁵ m⁻¹)

Therefore,

$$-\frac{dP}{\frac{A}{P}+BP}=dx$$

Integrating from x=0 to x=L gives

Note: Use Wolfram Integrator

$$\frac{1}{2B}Ln\left(\frac{A+BP_o^2}{A+BP_L^2}\right)=L$$

Solve using Goal Seek in Excel using

$$\mathrm{f(P_o)}=\frac{1}{2B}Ln\left(\frac{A+BP_o^2}{A+BP_L^2}\right)-L$$