

2.1 Refer to the results of Example 2.1. The viscosity of the glass is 1 N s m^{-2} , and the viscosity of the metal is $3 \times 10^{-3} \text{ N s m}^{-2}$. The densities of the glass and the metal are 3.2 kg m^{-3} and 7.0 kg m^{-3} , respectively. For $\beta = \pi/8$ and $\delta_1 = 1 \text{ mm}$ and $\delta_2 = 2 \text{ mm}$, calculate the maximum velocities and average velocities of the glass and the metal.

glass

$$v_z = -\frac{x^2 g \cos \beta}{2\nu_g} + c_1, \quad \delta_1 \leq x \leq 0$$

BB $c_2 = \frac{(\rho_g - \rho_m) \delta_1 g \cos \beta}{\eta_m}$,
 CC $c_3 = \frac{\delta_2^2 g \cos \beta}{2\nu_m} + c_2 \delta_2$,

metal

$$v_z = -\frac{x^2 g \cos \beta}{2\nu_m} - c_2 x + c_3, \quad \delta_1 \leq x \leq \delta_2$$

AA $c_1 = \left[\frac{\delta_2^2 - \delta_1^2}{\nu_m} + \frac{\delta_1^2}{\nu_g} \right] \frac{g \cos \beta}{2} + c_2 (\delta_2 - \delta_1)$

Parameters	check	use	check
Ng	1	BB	-11.4801 -11.4801 c2
Nm	0.003	CC	0.019335 0.019335 c3
pg	3.2	AA	0.020256 0.020256 c1
pm	7		
vg	3.13E-01		
vm	4.29E-04		
dg	0.001		
dm	0.002		
g	9.81		
B	0.393		

TIP
 In Excel
 $-3^2=9$
 $-(3^2)=-9$
 if $x=3$
 $-x^2=9$

x		v	vg	vm
m	mm			
0	0	0.020	0.02	
0.0002	0.2	0.020	0.02	
0.0004	0.4	0.020	0.02	
0.0006	0.6	0.020	0.02	
0.0008	0.8	0.020	0.02	
0.0010	1	0.020	0.02	0.02
0.0012	1.2	0.018		0.02
0.0014	1.4	0.015		0.01
0.0016	1.6	0.011		0.01
0.0018	1.8	0.006		0.01
0.0020	2	0.000		0.00

$$Q_g = W_g \int_0^{\delta_g} V_g(x) dx = \frac{W_g \delta_g^3 \rho_g g \cos(\beta)}{6\eta_g} + c_1 \delta_g$$

$$\bar{V}_g = \frac{Q_g}{\delta_g W_g} = \frac{\delta_g^3 \rho_g g \cos(\beta)}{6\eta_g} + c_1 = 2.03\text{E-}02 \text{ m}^3/\text{s} \quad \text{glass}$$

$$Q_m = W_m \int_{\delta_1}^{\delta_2} V_m(x) dx = \frac{W_m (\delta_m^3 - \delta_g^3) \rho_g g \cos(\beta)}{6\eta_g} - \frac{c_2 (\delta_m^2 - \delta_g^2)}{2} + c_3 (\delta_m - \delta_g)$$

$$\bar{V}_g = \frac{Q_m}{(\delta_m - \delta_g) W_m} = \frac{(\delta_m^3 - \delta_g^3) \rho_g g \cos(\beta)}{(\delta_m - \delta_g) 6\eta_m} - \frac{c_2 (\delta_m^2 - \delta_g^2)}{2(\delta_m - \delta_g)} + c_3 = 1.19\text{E-}02 \text{ m}^3/\text{s} \quad \text{metal}$$

- 2.5** Repeat Problem 2.4 but now orient the plates at an angle β to the direction of gravity and obtain expressions for
- the velocity distribution,
 - the volume flow rate.
- Compare your expressions with the results of Problem 2.4 and Eqs. (2.20) and (2.23).

2.4 Develop expressions for the flow of a fluid between vertical parallel plates. The plates are separated by a distance of 2δ . Consider fully developed laminar flow.

are separated by a distance of $2b$. Consider fully developed flow and determine

- the velocity distribution,
- the volume flow rate.

Compare your expressions with Eqs. (2.20) and (2.23).

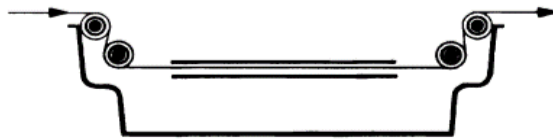
Simply add $\rho g \cos(B)$ to the $(P_0 - P_L)/L$ term

2.6 A liquid is flowing through a vertical tube 0.3 m long and 2.5 mm in I.D. The density of the liquid is 1260 kg m^{-3} and the mass flow rate is $3.8 \times 10^{-5} \text{ kg s}^{-1}$.

- What is the viscosity in N s m^{-2} ?
- Check on the validity of your results.

radius	0.00125 m	$Q = \left[\frac{P_0 - P_L}{L} + \rho g \right] \left[\frac{\pi R^4}{8\eta} \right]$	(2.34)
density	1260 kg/m^3		
Mrate	3.80E-05 kg/s		
Q	3.02E-08 m^3/s		
g	9.81 m/s^2	Vav	0.006144 m/s
Vis	0.393 kg/m/s	Re	0.049252 < 2000 laminar

2.10 A wire is cooled after a heat treating operation by being pulled through the center of an open-ended, oil-filled tube which is immersed in a tank. In a region in the tube where end effects are negligible, obtain an expression for the velocity profile assuming steady state and all physical properties constant.



Tube inner radius: R
Wire radius: KR
Wire velocity: U

$$\frac{d(r \tau_{rz})}{dr} = 0$$

$$r \tau_{rz} = C_1$$

$$-\eta \frac{dv_z}{dr} = C_1/r$$

$$v_z = -C_1 \ln r + C_2$$

Apply BC's

$$0 = -C_1 \ln R + C_2$$

$$+ U = -C_1 \ln KR + C_2$$

$$U = -C_1 \ln K$$

$$C_1 = -U / \ln K$$

$$C_2 = -\frac{U}{\ln K} \ln R$$

$$\therefore v_z = \frac{U}{\ln K} \ln(r/R)$$