

2.1 Refer to the results of Example 2.1. The viscosity of the glass is 1 N s m^{-2} , and the viscosity of the metal is $3 \times 10^{-3} \text{ N s m}^{-2}$. The densities of the glass and the metal are 3.2 kg m^{-3} and 7.0 kg m^{-3} , respectively. For $\beta = \pi/8$ and $\delta_1 = 1 \text{ mm}$ and $\delta_2 = 2 \text{ mm}$, calculate the maximum velocities and average velocities of the glass and the metal.

glass

$$u_z = -\frac{x^2 g \cos \beta}{2\nu_g} + c_1, \quad \delta_1 \leq x \leq 0$$

metal

$$u_z = -\frac{x^2 g \cos \beta}{2\nu_m} - c_2 x + c_3, \quad \delta_1 \leq x \leq \delta_2$$

$$\text{BB} \quad c_2 = \frac{(\rho_g - \rho_m) \delta_1 g \cos \beta}{\eta_m}$$

$$\text{CC} \quad c_3 = \frac{\delta_2^2 g \cos \beta}{2\nu_m} + c_2 \delta_2$$

$$\text{AA} \quad c_1 = \left[\frac{\delta_2^2 - \delta_1^2}{\nu_m} + \frac{\delta_1^2}{\nu_g} \right] \frac{g \cos \beta}{2} + c_2 (\delta_2 - \delta_1)$$

Parameters

| | |
|----|---------|
| Ng | 1 |
| Nm | 0.003 |
| pg | 3.2 |
| pm | 7 |
| vg | =Ng/pg |
| vm | =Nm/pm |
| dg | 0.001 |
| dm | 0.002 |
| g | =g |
| B | =PI()/8 |

check

| |
|-----|
| =Ng |
| =Nm |
| =pg |
| =pm |
| =vg |
| =vm |
| =dg |
| =dm |
| =g |
| =B |

use

| | |
|----|-------------------------|
| BB | =(pg-pm)*dg*g*COS(B)=BB |
| CC | =dm^2*g*COS(B)/2/vn=CC |
| AA | =(dm^2-dg^2)/vm+d1=AA |

check

| |
|----|
| c2 |
| c3 |
| c1 |

TIP
In Excel
-3^2=9
-(3^2)=-9
if x=3
-x^2=9

| x | | v | vg | vm |
|-------------|-----------|------|---------------------|---------------------|
| m | mm | | | |
| 0 | =E22*1000 | =H22 | =(E22^2)*g*COS(B)/2 | |
| =E22+0.0002 | =E23*1000 | =H23 | =(E23^2)*g*COS(B)/2 | |
| =E23+0.0002 | =E24*1000 | =H24 | =(E24^2)*g*COS(B)/2 | |
| =E24+0.0002 | =E25*1000 | =H25 | =(E25^2)*g*COS(B)/2 | |
| =E25+0.0002 | =E26*1000 | =H26 | =(E26^2)*g*COS(B)/2 | |
| =E26+0.0002 | =E27*1000 | =H27 | =(E27^2)*g*COS(B)/2 | =(E27^2)*g*COS(B)/2 |
| =E27+0.0002 | =E28*1000 | =I28 | | =(E28^2)*g*COS(B)/2 |
| =E28+0.0002 | =E29*1000 | =I29 | | =(E29^2)*g*COS(B)/2 |
| =E29+0.0002 | =E30*1000 | =I30 | | =(E30^2)*g*COS(B)/2 |
| =E30+0.0002 | =E31*1000 | =I31 | | =(E31^2)*g*COS(B)/2 |
| =E31+0.0002 | =E32*1000 | =I32 | | =(E32^2)*g*COS(B)/2 |

$$Q_g = W_g \int_0^{\delta_g} V_g(x) dx = \frac{W_g \delta_g^3 \rho_g g \cos(\beta)}{6\eta_g} + c_1 \delta_g$$

$$\bar{V}_g = \frac{Q_g}{\delta_g W_g} = \frac{\delta_g^3 \rho_g g \cos(\beta)}{6\eta_g} + c_1$$

glass
=-dg^2*pg*g*COS(B)/6 m^3/s

$$Q_m = W_m \int_{\delta_1}^{\delta_2} V_m(x) dx = \frac{W_m (\delta_m^3 - \delta_g^3) \rho_g g \cos(\beta)}{6\eta_m} - \frac{c_2 (\delta_m^2 - \delta_g^2)}{2} + c_3 (\delta_m - \delta_g)$$

$$\bar{V}_g = \frac{Q_m}{(\delta_m - \delta_g) W_m} = \frac{(\delta_m^3 - \delta_g^3) \rho_g g \cos(\beta)}{(\delta_m - \delta_g) 6\eta_m} - \frac{c_2 (\delta_m^2 - \delta_g^2)}{2(\delta_m - \delta_g)} + c_3$$

metal
=-(dm^3-dg^3)*pm*g*cos(B)/6 m^3/s

2.5 Repeat Problem 2.4 but now orient the plates at an angle β to the direction of gravity and obtain expressions for

- the velocity distribution,
- the volume flow rate.

Compare your expressions with the results of Problem 2.4 and Eqs. (2.20) and (2.23).

2.4 Develop expressions for the flow of a fluid between vertical parallel plates. The plates are separated by a distance of 2δ . Consider fully developed flow and determine

- the velocity distribution,
- the volume flow rate.

Compare your expressions with Eqs. (2.20) and (2.23).

Simply add pgCos(B) to the (Po=PL)/L term

2.6 A liquid is flowing through a vertical tube 0.3 m long and 2.5 mm in I.D. The density of the liquid is 1260 kg m⁻³ and the mass flow rate is 3.8 × 10⁻⁵ kg s⁻¹.

- a) What is the viscosity in N s m⁻²?
 b) Check on the validity of your results.

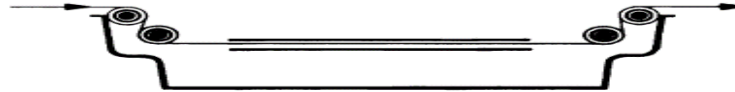
| | | |
|---------|------------------------------|-------------------|
| radius | =0.0025/2 | m |
| density | 1260 | kg/m ³ |
| Mrate | 0.000038 | kg/s |
| Q | =Mrate/density | m ³ /s |
| g | 9.81 | m/s ² |
| Vis | =density*g*PI()*radius^4/8/Q | kg/m/s |

$$Q = \left[\frac{P_o - P_L}{L} + \rho g \right] \left[\frac{\pi R^4}{8\eta} \right] \quad (2.34)$$

Vav = Q/(PI()*radius^2) m/s
 Re = density*G68*2*radius < 2000 laminar

2.10 A wire is cooled after a heat treating operation by being pulled through the center of an open-ended, oil-filled tube which is immersed in a tank. In a region in the tube where end effects are negligible, obtain an expression for the velocity profile assuming steady state and all physical properties constant.

Tube inner radius: R
 Wire radius: KR
 Wire velocity: U



$$\frac{d(\sigma r z)}{dr} = 0$$

$$r z z = C_1$$

$$-r \frac{dv_z}{dr} = C_1/r$$

$$v_z = -C_1 \ln r + C_2$$

Apply BC'S

$$0 = -C_1 \ln R + C_2$$

$$+ U = -C_1 \ln KR + C_2$$

$$U = -C_1 \ln K$$

$$C_1 = U / \ln K$$

$$C_2 = -\frac{U}{\ln K} \ln R$$

$$\therefore v_z = \frac{U}{\ln K} \ln (r/R)$$