**2.1** Refer to the results of Example 2.1. The viscosity of the glass is 1 N s m$^{-2}$, and the viscosity of the metal is 3 × 10$^{-3}$ N s m$^{-2}$. The densities of the glass and the metal are 3.2 kg m$^{-3}$ and 7.0 kg m$^{-3}$, respectively. For $\beta = \pi/8$ and $\delta_1 = 1$ mm and $\delta_2 = 2$ mm, calculate the maximum velocities and average velocities of the glass and the metal.

**glass**

$$v_z = -\frac{x^2 g \cos\beta}{2\nu_g} + c_1, \qquad \delta_1 \le x \le 0$$

**metal**

$$v_z = -\frac{x^2 g \cos\beta}{2\nu_m} - c_2 x + c_3, \qquad \delta_1 \le x \le \delta_2$$

BB $$c_2 = \frac{(\rho_g - \rho_m)\, \delta_1\, g \cos\beta}{\eta_m},$$

CC $$c_3 = \frac{\delta_2^2\, g \cos\beta}{2\nu_m} + c_2\, \delta_2,$$

AA $$c_1 = \left[\frac{\delta_2^2 - \delta_1^2}{\nu_m} + \frac{\delta_1^2}{\nu_g}\right]\frac{g \cos\beta}{2} + c_2(\delta_2 - \delta_1)$$

| Parameters | | check |
|---|---|---|
| Ng | 1 | 1 |
| Nm | 0.003 | 0.003 |
| pg | 3.2 | 3.2 |
| pm | 7 | 7 |
| vg | 3.13E-01 | 3.13E-01 |
| vm | 4.29E-04 | 4.29E-04 |
| dg | 0.001 | 0.001 |
| dm | 0.002 | 0.002 |
| g | 9.81 | 9.81 |
| B | 0.393 | 0.393 |

| use | | check | |
|---|---|---|---|
| BB | -11.4801 | -11.4801 | c2 |
| CC | 0.019335 | 0.019335 | c3 |
| AA | 0.020256 | 0.020256 | c1 |

**TIP**
In Excel
-3^2 =9
-(3^2)=-9
if x=3
-x^2 =9

| x | | v | vg | vm |
|---|---|---|---|---|
| m | mm | m/s | m/s | m/s |
| 0 | 0 | 0.020 | 0.02 | |
| 0.0002 | 0.2 | 0.020 | 0.02 | |
| 0.0004 | 0.4 | 0.020 | 0.02 | |
| 0.0006 | 0.6 | 0.020 | 0.02 | |
| 0.0008 | 0.8 | 0.020 | 0.02 | |
| 0.0010 | 1 | 0.020 | 0.02 | 0.02 |
| 0.0012 | 1.2 | 0.018 | | 0.02 |
| 0.0014 | 1.4 | 0.015 | | 0.01 |
| 0.0016 | 1.6 | 0.011 | | 0.01 |
| 0.0018 | 1.8 | 0.006 | | 0.01 |
| 0.0020 | 2 | 0.000 | | 0.00 |

$$Q_g = W_g \int_0^{\delta_g} V_g(x)dx = \frac{W_g \delta_g^3 \rho_g g \cos(\beta)}{6\eta_g} + c_1\delta_g$$

$$\overline{V}_g = \frac{Q_g}{\delta_g W_g} = \frac{\delta_g^3 \rho_g g \cos(\beta)}{6\eta_g} + c_1$$

glass

= 0.02 m$^3$/s

$$Q_m = W_m \int_{\delta_1}^{\delta_2} V_m(x)dx = \frac{W_m\left(\delta_m^3 - \delta_g^3\right)\rho_g g \cos(\beta)}{6\eta_g} - \frac{c_2\left(\delta_m^2 - \delta_g^2\right)}{2} + c_3\left(\delta_m - \delta_g\right)$$

$$\overline{V}_g = \frac{Q_m}{\left(\delta_m - \delta_g\right)W_m} = \frac{\left(\delta_m^3 - \delta_g^3\right)\rho_g g \cos(\beta)}{\left(\delta_m - \delta_g\right)6\eta_m} - \frac{c_2\left(\delta_m^2 - \delta_g^2\right)}{2\left(\delta_m - \delta_g\right)} + c_3$$

metal

= 0.01 m$^3$/s

**2.5** Repeat Problem 2.4 but now orient the plates at an angle $\beta$ to the direction of gravity and obtain expressions for

a) the velocity distribution,
b) the volume flow rate.

Compare your expressions with the results of Problem 2.4 and Eqs. (2.20) and (2.23).

**2.4** Develop expressions for the flow of a fluid between vertical parallel plates. The plates are separated by a distance of 2δ. Consider fully developed flow ...

are separated by a distance of 2δ. Consider fully developed flow and determine
a) the velocity distribution,
b) the volume flow rate.
Compare your expressions with Eqs. (2.20) and (2.23).

Simply add pgCos(B) to the (Po=PL)/L term

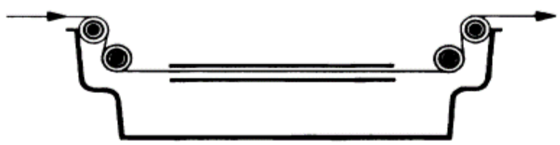
**2.6** A liquid is flowing through a vertical tube 0.3 m long and 2.5 mm in I.D. The density of the liquid is 1260 kg m$^{-3}$ and the mass flow rate is $3.8 \times 10^{-5}$ kg s$^{-1}$.
a) What is the viscosity in N s m$^{-2}$?
b) Check on the validity of your results.

$$Q = \left[\frac{P_0 - P_L}{L} + \rho g\right]\left[\frac{\pi R^4}{8\eta}\right]. \quad (2.34)$$

| | | | | | | |
|---|---|---|---|---|---|---|
| radius | 0.00125 | m | | | | |
| density | 1260 | kg/m$^3$ | | | | |
| Mrate | 3.80E-05 | kg/s | | | | |
| Q | 3.02E-08 | m$^3$/s | | | | |
| g | 9.81 | m/s$^2$ | Vav | 0.006144 | m/s | |
| Vis | 0.393 | kg/m/s | Re | 0.049252 | <2000 | laminar |

**2.10** A wire is cooled after a heat treating operation by being pulled through the center of an open-ended, oil-filled tube which is immersed in a tank. In a region in the tube where end effects are negligible, obtain an expression for the velocity profile assuming steady state and all physical properties constant.

Tube inner radius: *R*
Wire radius: *KR*
Wire velocity: *U*



$$\frac{d(r\tau_{rz})}{dr} = 0$$

$$r\tau_{rz} = C_1$$

$$-\eta \frac{dv_z}{dr} = C_1/r$$

$$v_z = -C_1 \ln r + C_2$$

Apply BC's

$$-\quad 0 = -C_1 \ln R + C_2$$

$$+\quad U = -C_1 \ln KR + C_2$$

$$U = C_1 \ln K$$

$$C_1 = U/\ln K$$

$$C_2 = \frac{U}{\ln K} \ln R$$

$$\therefore v_z = \frac{U}{\ln K} \ln(r/R)$$