SOUTH DAKOTA SCHOOL OF MINES AND TECHNOLOGY
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Met 422 Homework 05 Solution to Problem 2 10/21

2) Convert Eq (A) in Table 7.5 to dimensionless form and show the emergence of the Re, Prandtl (Pr), and Brinkman (Br) numbers.

Table 7.5 The equation of energy in terms of the transport properties (*for Newtonian fluids of constant ρ, η, and k; note that the constancy of ρ implies that $C_v = C_p$*)

Rectangular coordinates

$$\rho C_v\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right]$$
$$+ 2\eta\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \eta\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2\right.$$
$$\left. + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\}. \qquad \text{(A)}$$

Operate on the red-circled terms. All other terms within each bracket will convert to dimensionless form via the same variables as those circled.

$$\rho Cp\left(.+Vv_x^*\frac{\Delta T\partial\Theta}{D\partial x^*}+.\right)=k\left(\frac{\Delta T\partial^2\Theta}{D^2\partial x^{*2}}+.\right)+\eta\left(\left(\frac{V\partial v_x^*}{D\partial x^*}\right)^2+.\right)$$

$$\frac{\rho CpV\Delta T}{D}\left(.+v_x^*\frac{\partial\Theta}{\partial x^*}+.\right)=\frac{k\Delta T}{D^2}\left(\frac{\partial^2\Theta}{\partial x^{*2}}+.\right)+\frac{\eta V^2}{D^2}\left(\left(\frac{\partial v_x^*}{\partial x^*}+.\right)^2+.\right)$$

$$\frac{\rho CpV\Delta T}{D}[\ ]=\frac{k\Delta T}{D^2}[\ ]+\frac{\eta V^2}{D^2}[\ ]$$

where $\Theta=\frac{T-T_o}{T_f-T_o}=\frac{T-T_o}{\Delta T}$

Brackets are now dimensionless. Divide by the first term and multiply and divide by the over-stroked pairs to give

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$$[\ ] = \frac{\bar{\eta}}{\rho VD}\frac{k}{\bar{\eta}Cp}[\ ] + \frac{\eta}{\rho VD}\frac{\bar{\eta}V^2}{\bar{k}\Delta T}\frac{\bar{k}}{\bar{\eta}Cp}[\ ]$$

Which may be re-arranged to give

$$[\ ] = \frac{1}{\mathrm{Re}}\frac{1}{\mathrm{Pr}}[\ ] + \frac{1}{\mathrm{Re}}\frac{1}{Br}\frac{1}{\mathrm{Pr}}[\ ]$$

The Brinkmann #

$$Br = \frac{\eta V^2}{\bar{k}\Delta T}[=]\frac{\frac{kg}{m*s}*\frac{m^2}{s^2}}{\frac{kg*m^2}{s^2*m*K}*K}[=]none$$

is used to model plastic flow such as in extrusion of polymers and friction stir welding.