Homework 03B

3.10 Packed Bed Solution for compressible fluids (gases)

Assume

Flow is top down from x=0 to x=L

M = mass flow rate, given

Po = inlet pressure at x=0, find

PL = outlet pressure at x=L, given

ρ = density of incoming gas, given

T = constant gas temperature in K, given

A = Unrestricted cross-sectional bed area, given

R = Gas Constant 0.08205 atm*m³/(kg mol*K) = 0.08205/101325 pa*m³/(kg mol*K)

MW = Molecular weight in kg/kg mole, not given

$T := 800 \cdot K$ $\quad RR := (8.314 \cdot 10^3) \frac{m^3 \cdot Pa}{K \cdot mol}$ $\quad PL := 1.4 \cdot 10^5 \cdot Pa$ $\quad Area := \pi \cdot 3^2 \cdot m^2$

$MW := \frac{0.5 \cdot \frac{kg}{m^3} \cdot RR \cdot T}{1 \cdot atm} = 32.821 \frac{kg}{mol}$ $\quad L := 15 \cdot m$ $\quad \rho(P) := 1 \cdot atm \cdot \frac{MW}{RR \cdot T}$ $\quad \rho(1\ atm) = ($

$\eta := 4.13 \cdot 10^{-5} \cdot \frac{kg}{m \cdot s}$ $\quad \omega := 0.4$ $\quad M := 95 \cdot \frac{kg}{s}$ $\quad Dp := 0.003 \cdot m$

$$A := \frac{150 \cdot \eta \cdot (1-\omega)^2 \cdot RR \cdot T \cdot M}{Dp^2 \cdot \omega^3 \cdot Area \cdot MW} + \frac{1.75 \cdot (1-\omega) \cdot M^2 \cdot RR \cdot T}{Dp \cdot \omega^3 \cdot Area^2 \cdot MW} = (1.515 \cdot 10^{10}) \frac{kg^2}{m^3 \cdot s^4}$$

$B := -\left(\frac{MW}{RR \cdot T} \cdot g\right) = -4.839 \cdot 10^{-5} \frac{1}{m}$ $\quad f(Po) := \frac{1}{2 \cdot B} \cdot \ln\left(\frac{A + B \cdot Po^2}{A + B \cdot PL^2}\right) - L$

Using MathCad using ***INSERTED*** roots function under menu "Functions/Solving"

Guess $\quad PoGuess := 2 \cdot PL$ $\quad RootPo := \text{root}\left(f(PoGuess), PoGuess, 0, 10^6\right)$

$RootPo = 6.882 \cdot 10^5$ **Ans**

BY Trial and Error : guess upper limit. Ans 4.916 times Po is when the LHS of the integral equals L.

$$ff(P) := \int_{PL}^{PL \cdot 4.916} \frac{1}{\frac{A}{P} + B \cdot P} \, \mathrm{d}P = 15 \; m$$

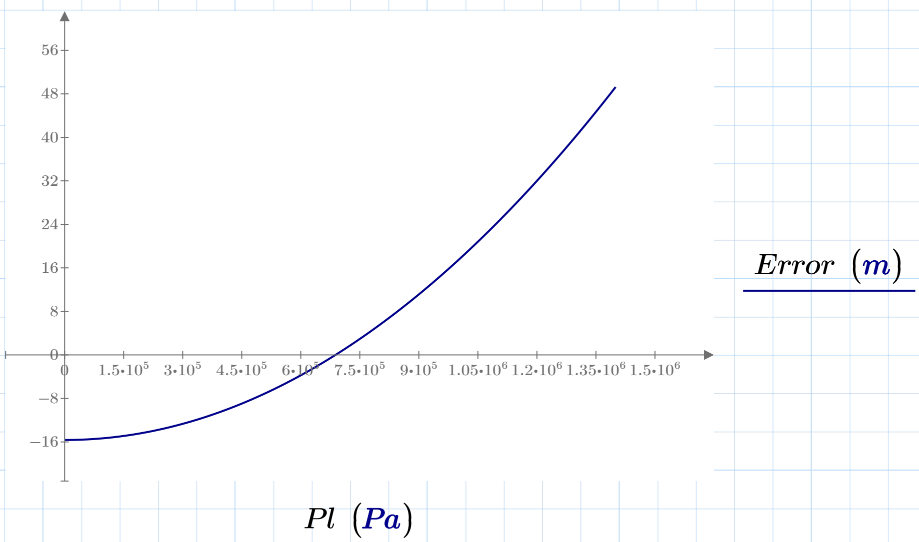
root(f(var1, var2, ...), var1, [a, b])—Returns the value of var1 to make the function f equal to zero. If a and b are specified, root finds var1 on the interval [a, b]. Otherwise, var1 must be defined with a guess value before root is called. When a guess value is used, root uses the secant or Mueller's method; in the case where root bracketing is used, root uses Ridder's or Brent's method.

Plot

$$i := 1, 2 .. 100$$

$$Pl_i := \frac{i}{10} \cdot PL$$

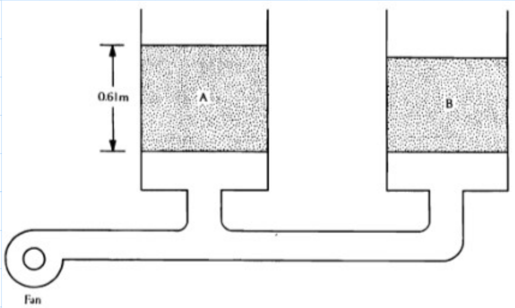
$$Error := f(Pl)$$



3.21 A fan delivers air to two fluidized beds, *A* and *B*. Bed *A* is operating at a minimum volume (i.e., at minimum fluidization) and bed *B* is fluidized to a volume equal to twice its fixed bed volume.

a) Calculate the superficial velocity through bed *A*.
b) Calculate the superficial velocity through bed *B*.
c) Calculate ΔP across bed *A*.
d) For bed *B*, prove that $\omega = 0.7$ when it is fluidized to twice the fixed bed volume.

Bed *A*: $D_p = 91.4\ \mu m$ (uniform); ρ (solid) $= 4808\ kg\ m^{-3}$; $\lambda = 1$.
Bed *B*: $D_p = 61.0\ \mu m$ (uniform); ρ (solid) $= 4006\ kg\ m^{-3}$; $\lambda = 1$;
ω (fixed bed) $= 0.4$; ω (fluidized) $= 0.7$.
Air: $\rho = 1.28\ kg\ m^{-3}$; $\eta = 2.07 \times 10^{-5}\ N\ s\ m^{-2}$.



$$\left[\frac{Re}{f}\right]^{1/3} = \frac{V_0}{\left[\frac{4g\eta_f(\rho_p - \rho_f)}{3\rho_f^2}\right]^{1/3}}$$

$$(f Re^2)^{1/3} = \frac{D_p}{\left[\frac{3\eta_f^2}{4g\rho_f(\rho_p - \rho_f)}\right]^{1/3}}.$$

a) $DpA := 91.4 \cdot \mu m \qquad \rho := 1.28 \cdot kg \cdot m^{-3} \qquad L := 0.61 \cdot m$

$\rho sA := 4808 \cdot kg \cdot m^{-3} \qquad \eta := 2.07 \cdot 10^{-5} \cdot kg \cdot m^{-1} \cdot s^{-1} \qquad \omega A := 0.4$

$$XDp := \frac{DpA}{\left(\frac{3 \cdot \eta^2}{4 \cdot g \cdot \rho \cdot (\rho sA - \rho)}\right)^{\frac{1}{3}}} = 5.234 \qquad YVo := 0.063$$

$$Vo := 0.063 \cdot \left(\frac{4 \cdot g \cdot \eta \cdot (\rho sA - \rho)}{3 \cdot \rho^2}\right)^{\frac{1}{3}} = 0.058\ \frac{m}{s}$$

b) $DpB := 61 \cdot \mu m \qquad \rho := 1.28 \cdot kg \cdot m^{-3}$

$\rho sB := 4006 \cdot kg \cdot m^{-3} \qquad \eta := 2.07 \cdot 10^{-5} \cdot kg \cdot m^{-1} \cdot s^{-1} \qquad \omega B := 0.7$

$$XDp := \frac{DpB}{\left(\frac{3 \cdot \eta^2}{4 \cdot g \cdot \rho \cdot (\rho sA - \rho)}\right)^{\frac{1}{3}}} = 3.493 \qquad YVo := 0.06$$

$$Vo := 0.06 \cdot \left(\frac{4 \cdot g \cdot \eta \cdot (\rho sB - \rho)}{3 \cdot \rho^2}\right)^{\frac{1}{3}} = 0.052\ \frac{m}{s}$$

c) $\Delta P := \rho sA \cdot g \cdot (1 - \omega A) \cdot L = (1.726 \cdot 10^4)\ Pa$

d) $1 - \frac{(1 - 0.4)}{2} = 0.7$

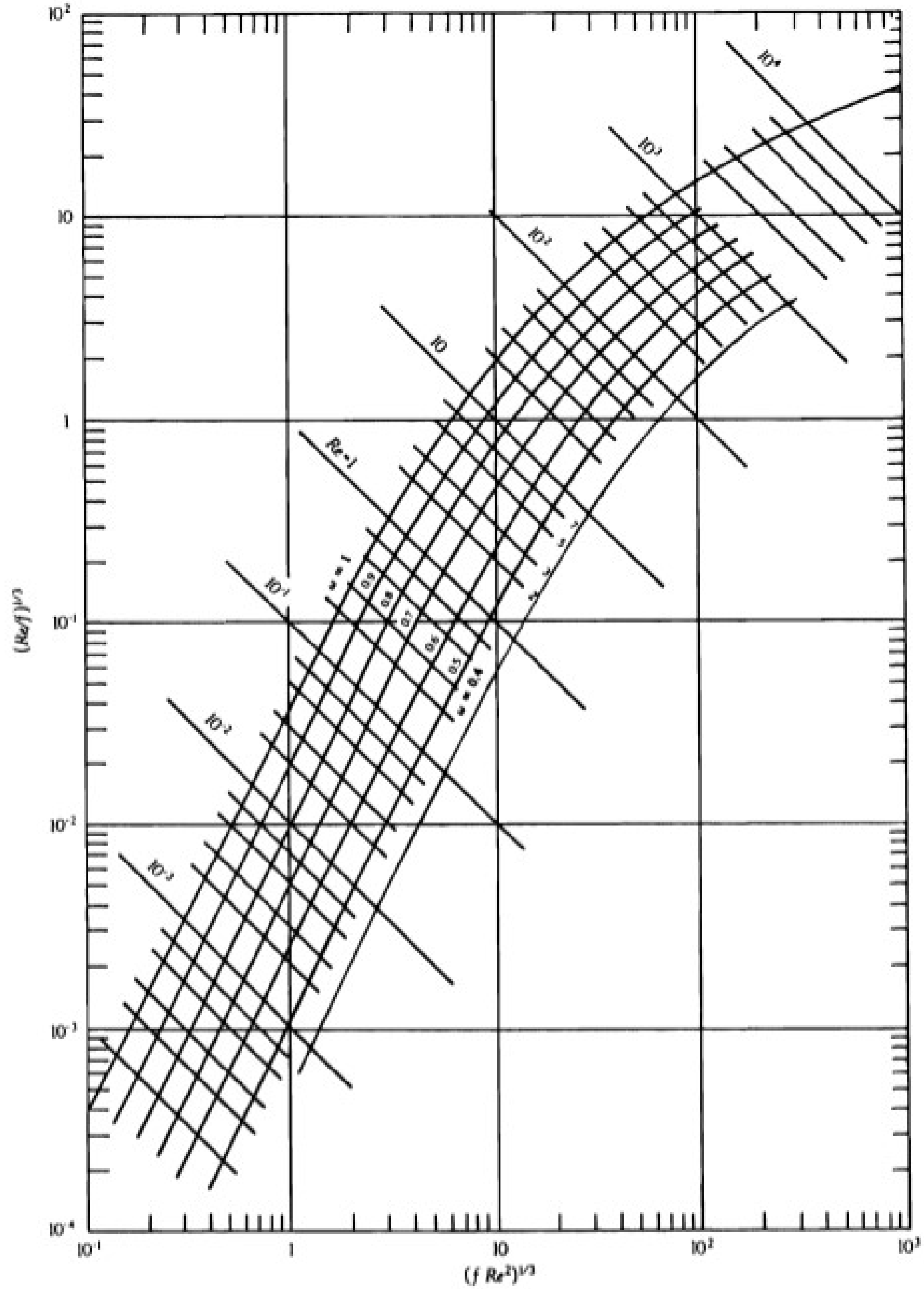


Fig 3.13 Smoothed correlation of particulate fluidization. (From F. A. Zenz and D. F. Othmer, *Fluidization and Fluid Particle Systems*, Reinhold Publ. Corp., New York, NY, 1960, page 236.)

3.23 Pellets of polyethylene are to be fluidized in a column 1 meter in diameter and 10 meters high with air at 300 K. A hot steel pipe is lowered into the fluidized bed to bring it into contact with the pellets, which melt onto its surface to form a protective anticorrosion coating. Calculate the total flow of air required, if the pellets are 5 mm in diameter and the desired void fraction of the bed is 0.7. The density of the polyethylene is 920 kg m^{-3}.

$$Dp := 5 \cdot mm \qquad \rho := \frac{1\ atm \cdot 28.6 \cdot \frac{kg}{mole}}{0.08205 \cdot \frac{atm \cdot m^3}{mol \cdot K} \cdot 300 \cdot K} = 1.162\ \frac{kg}{m^3} \qquad \omega := 0.7$$

$$L := 10 \cdot m \qquad D := 1 \cdot m \qquad \rho s := 920 \cdot kg \cdot m^{-3} \qquad \eta := 1.846 \cdot 10^{-5} \cdot kg \cdot m^{-1} \cdot s^{-1}$$

$$XDp := \frac{Dp}{\left(\frac{3 \cdot \eta^2}{4 \cdot g \cdot \rho \cdot (\rho s - \rho)}\right)^{\frac{1}{3}}} = 172.36 \qquad YVo := 8.0$$

$$Vo := 8.0 \cdot \left(\frac{4 \cdot g \cdot \eta \cdot (\rho s - \rho)}{3 \cdot \rho^2}\right)^{\frac{1}{3}} = 4.381\ \frac{m}{s}$$

$$Area := \pi \cdot \left(\frac{D}{2}\right)^2 \qquad Q := Area \cdot Vo = 3.441\ \frac{m^3}{s} \qquad W := Q \cdot \rho = 3.998\ \frac{kg}{s}$$

4. For a bed of 1 cm diameter marbles with a density of 2.7 g/cm^3,
   a) What is the minimum fluidization velocity to fluidize the bed?
   b) What is the V_o to make the void fraction 1?
   c) What is the V_o to make the void fraction 0.6?

Assume dry air at 500 K is being used.

a) $Dp := 1 \cdot cm$ $\quad \rho air := \dfrac{1\ atm \cdot 28.6 \cdot \dfrac{kg}{mole}}{0.08205 \cdot \dfrac{atm \cdot m^3}{mol \cdot K} \cdot 500 \cdot K} = 0.697\ \dfrac{kg}{m^3}$ $\quad L := 0.61 \cdot m$

$\rho s := 2700 \cdot kg \cdot m^{-3}$ $\quad \eta := 2.70 \cdot 10^{-5} \cdot kg \cdot m^{-1} \cdot s^{-1}$ $\quad \omega := 0.4$

$$XDp := \frac{Dp}{\left(\dfrac{3 \cdot \eta^2}{4 \cdot g \cdot \rho \cdot (\rho s - \rho air)}\right)^{\frac{1}{3}}} = 383.163 \qquad YVo := 4.0$$

$$Vo := 4.0 \cdot \left(\frac{4 \cdot g \cdot \eta \cdot (\rho s - \rho air)}{3 \cdot \rho air^2}\right)^{\frac{1}{3}} = 5.007\ \frac{m}{s}$$

b)

$YVo := 30$

$$Vo := 30 \cdot \left(\frac{4 \cdot g \cdot \eta \cdot (\rho s - \rho air)}{3 \cdot \rho air^2}\right)^{\frac{1}{3}} = 37.549\ \frac{m}{s}$$

c)

$YVo := 9.0$

$$Vo := 9.0 \cdot \left(\frac{4 \cdot g \cdot \eta \cdot (\rho s - \rho air)}{3 \cdot \rho air^2}\right)^{\frac{1}{3}} = 11.265\ \frac{m}{s}$$