3.1 Water at 300 K is flowing through a brass tube that is 30.0 m long and 13 mm in diameter (inner). The water is moving through the tube at a rate of 3.2×10^{-3} m^3 s^{-1}. The density of water is 1000 kg m^{-3}, and its viscosity is 8.55×10^{-4} N s m^{-2}. Calculate the pressure drop in Pa that accompanies this flow.

$$D := 13\ mm \qquad L := 30 \cdot m \qquad \rho := 1000 \cdot \frac{kg}{m^3} \qquad \eta := 8.55 \cdot 10^{-4} \cdot \frac{N \cdot s}{m^2}$$

$$R := \frac{D}{2} = 0.007\ m \qquad Q := 3.2 \cdot 10^{-3} \cdot \frac{m^3}{s} \qquad V := \frac{Q}{\pi \cdot R^2} = 24.109\ \frac{m}{s}$$

$$Re := \frac{D \cdot \rho \cdot V}{\eta} = 3.666 \cdot 10^5$$ From Figuure 3.2: $f := 0.004$

$$A_w := \pi \cdot D \cdot L = 1.225\ m^2 \qquad K := \frac{1}{2} \cdot \rho \cdot V^2 = \left(2.906 \cdot 10^5\right) \frac{N}{m^2}$$

$$F_k := f \cdot A_w \cdot K = \left(1.424 \cdot 10^3\right) N$$

Ans: $$\Delta P := \frac{F_k}{\pi \cdot R^2} = \left(1.073 \cdot 10^7\right) Pa \qquad \Delta P = \left(1.556 \cdot 10^3\right) psi$$
$$\Delta P = 105.901\ atm$$

Inappropriately assuming laminar flow and usng the Hagen-Poiseuille Eq. gives

$$\Delta P(R,\eta,V,Q,L,\rho) := \frac{Q \cdot 8 \cdot \eta \cdot L}{\pi \cdot R^4} \qquad \Delta P(R,\eta,V,Q,L,\rho) = 16.983\ psi$$

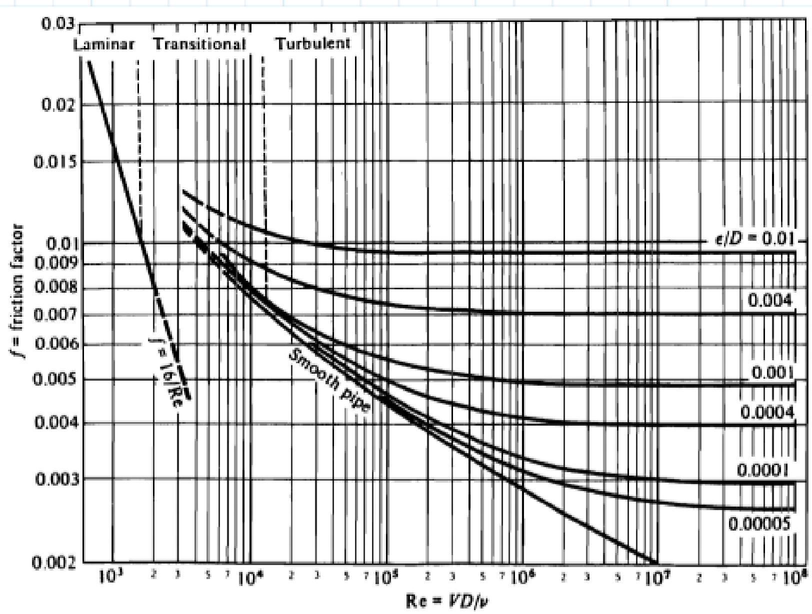

Fig. 3.2 Friction factors for flow in tubes. (Adapted from L. F. Moody, *Trans. ASME* **66**, 671 (1944), and *Mech. Eng.* **69**, 1005 (1947).)

3.5 Determine the size of the largest alumina particle in Problem 2.14 that would be expected to obey Stokes' law, remembering that for spheres this law is valid for Re $\leq$ 1.

2.14 In steelmaking, deoxidation of the melt is accomplished by the addition of aluminum, which combines with the free oxygen to form alumina, Al_2O_3. It is then hoped that most of these alumina particles will float up to the slag layer for easy removal from the process, because their presence in steel can be detrimental to mechanical properties. Determine the size of the smallest alumina particles that will reach the slag layer from the bottom of the steel two minutes after the steel is deoxidized. It may be assumed that the alumina particles are spherical in nature. For the purpose of estimating the steel's viscosity use the data for Fe-0.5 wt pct C in Fig. 1.11. *Data*: Temperature of steel melt: 1873 K; steel melt depth: 1.5 m; density of steel: 7600 kg m^{-3}; density of alumina: 3320 kg m^{-3}.

$$\rho_{Fe} := 7600 \cdot \frac{kg}{m^3} \quad \rho_A := 3320 \cdot \frac{kg}{m^3} \quad \eta_{Fe} := 6.4 \cdot 10^{-3} \cdot \frac{kg}{s \cdot m} \quad Re := 1$$

Force Balance for the upward migrating Al2O3 particle is f=Fg/AK where

Fg = ((4/3)*pi*R^3*(pFe-pS)*g and

f = 16/Re = 16 since Re=1 for max size observing laminar flow

from laminar flow. Therefore, 16AK=Fg, which is re-arranged to solve for D.

The unknown V is taken from Re=1=DVp/viscosity or V=viscosity/(p*D).

Therefore

$$D := \left(\frac{12 \cdot \eta_{Fe}^{2}}{\rho_A \cdot (\rho_{Fe} - \rho_A) \cdot g} \right)^{\frac{1}{3}} = 0.152\ mm$$

3.6 A falling-sphere viscometer was used to determine the viscosity of a slag intended for the production of copper. The viscosity of the slag was determined to be 441.2 Poise, using a steel ball as the falling sphere. Is this a valid viscosity? Why or why not? If not, determine the real value of the viscosity and then calculate its kinematic viscosity. The density of the slag may be taken as one-half that of the steel ball.

Data: Radius of steel ball, 88.7 mm; terminal velocity of steel ball, 1.52 m s^{-1}.

$R := 88.7 \cdot mm$ $\quad D := 2 \cdot R$ $\qquad \rho_{Fe} := 7880 \cdot \frac{kg}{m^3}$ $\qquad \rho_S := 0.5 \cdot \rho_{Fe}$ $\qquad V_\infty := 1.52 \cdot \frac{m}{s}$

From the force balance on the steel ball consisting of weight ,W, bouyancy, B, and drag, Fk = W - B since Fk operates in the directon opposite the weight but with the bouyancy. Therefore, W - B = fAK. solving for f gives

$$f := \frac{\frac{4}{3} \cdot \pi \cdot R^3 \left(\rho_{Fe} - \rho_S\right) \cdot g}{\pi \cdot R^2 \frac{1}{2} \rho_S \cdot V_\infty^{\ 2}} = 1.004$$

From Fig 3.8 for f=0.5, Re = 100

$$\eta_S := \frac{D \cdot V_\infty \cdot \rho_S}{100} = 10.624 \frac{kg}{m \cdot s} \qquad \eta_S = 106.241 \ poise$$

Proposed η_S is wrong because it was computed from Stokes Law (i.e. laminar conditions Eq (2.121)

$$\eta_{S\_Stokes} := \frac{2}{9} \cdot \frac{R^2 \left(\rho_{Fe} - \rho_S\right) \cdot g}{V_\infty} = 444.435 \ poise$$

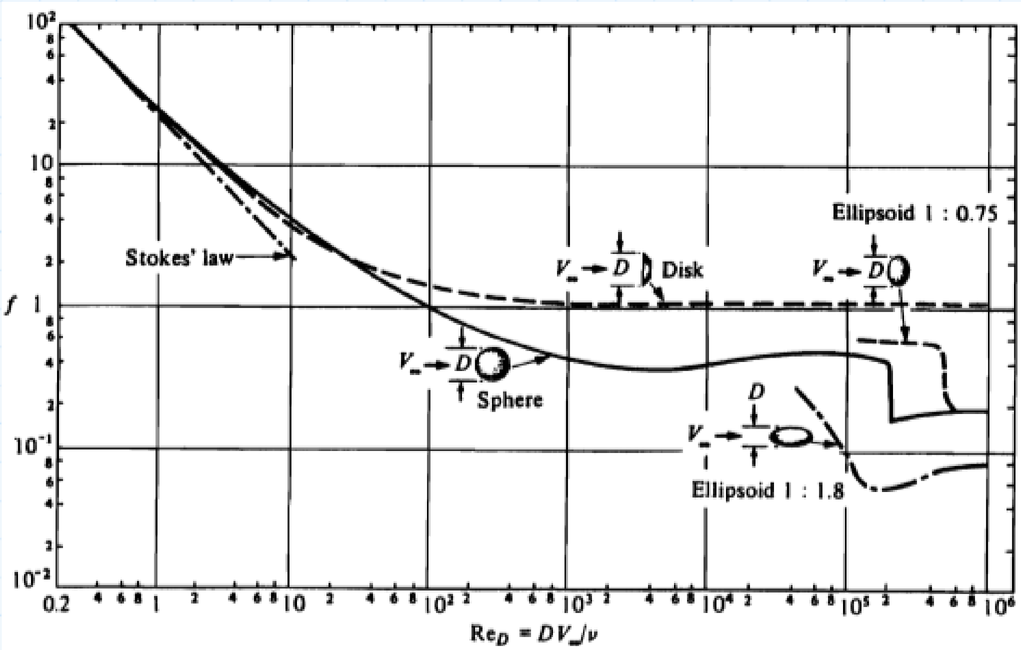

Fig. 3.8 Friction factors for submerged bodies. (Adapted from F. Eisner, *Proc. 3rd Intern. Congr. Appl. Mech.*, 1930, page 32.)

3.9 A thermocouple tube lies in a melt that is flowing perpendicular to the axis of the tube. Calculate the force per unit length of tube exerted by the flowing metal. *Data*: Velocity of the melt is 3 m s^{-1}; viscosity is 2×10^{-3} N s m^{-2}; density of the melt is 8000 kg m^{-3}; diameter of thermocouple tube is 61 mm.

$D := 61 \cdot mm$ $\quad L := 1 \cdot m$ $\quad A := D \cdot L$ $\quad \rho := 8000 \cdot \frac{kg}{m^3}$

$V := 3 \cdot \frac{m}{s}$ $\quad K := \frac{1}{2} \cdot \rho \cdot V^2$ $\quad \eta := 2 \cdot 10^{-3} \cdot \frac{N \cdot s}{m^2}$ $\quad Re := \frac{V \cdot D \cdot \rho}{\eta} = 7.32 \cdot 10^5$

From Fig 3.9, f =0.3

$Fk(f, A, K) := \frac{f \cdot A \cdot K}{L}$ $\quad Fk(0.3, A, K) = 658.8 \ \frac{N}{m}$

$Fk(0.3, A, K) = 45.142 \ \frac{lbf}{ft}$

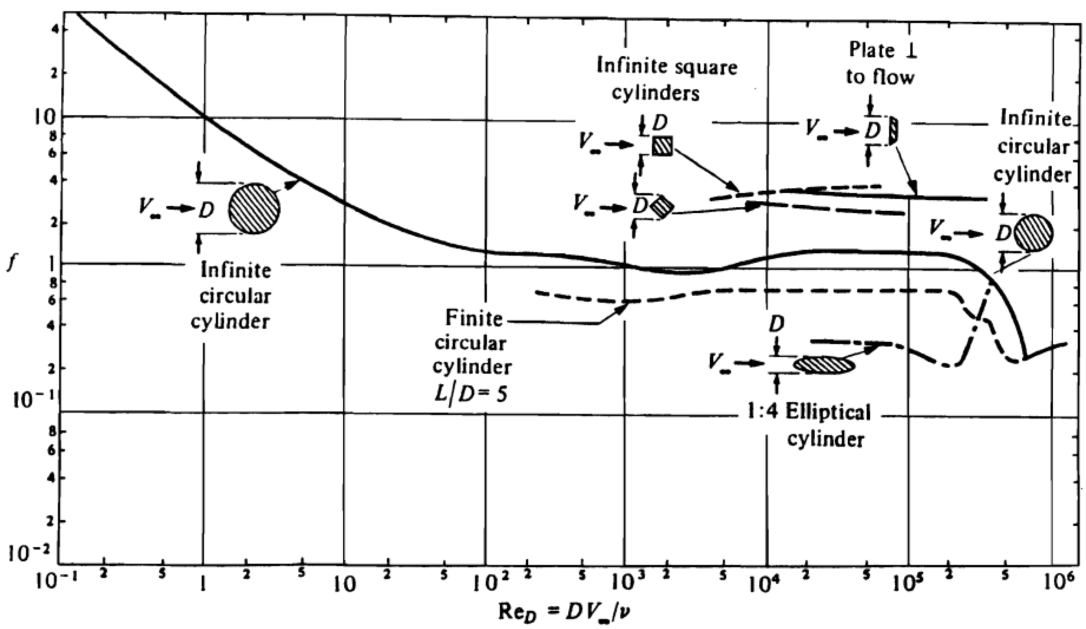

Fig. 3.9 Friction factors for submerged bodies. (Adapted from F. Eisner, *Proc. 3rd Intern. Congr. Appl. Mech.*, 1930, page 32.)