SOUTH DAKOTA SCHOOL OF MINES AND TECHNOLOGY

DEPARTMENT OF MATERIALS and METALLURGICAL ENGINEERING

MET 422 | Fluids | 02A Solution

1. Use the Equations of Change for Momentum.
   (This is ***like*** the problem worked as #1 in the videos Homework 02 Suppl A but you need to find which Eqs. of Change and which coordinate system to use.)

a) Obtain the ODE for steady-state laminar flow through a vertical cylinder with flow in the y-direction and r as the measure of radius. Both fluid viscosity and density are constant. There are the usual pressures P_o and P_L present.

b) How would the equation change if the tube were at an angle Beta to the direction of gravity?

c) Explain how the negative pressure gradient term is related to P_o and P_L

Table 2.3 The momentum equation in cylindrical coordinates (r, θ, z)

In terms of τ:

r-component*

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r \quad \text{(A)}$$

θ-component

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta \quad \text{(B)}$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \quad \text{(C)}$$

$-\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) - \frac{\partial P}{\partial z} + \rho g_z = 0$, same as derived since $-\frac{\partial P}{\partial z} = -\frac{P_L - P_o}{L - 0} = \frac{P_o - P_L}{L}$

If the tube were at an angle β to the vertical, $g_z = g\cos(\beta)$

2. Convert the first Equation of Change below into Dimensionless form and rearrange to get the Re and Fr numbers. (This is ***like*** the problem worked as #2 in videos Homework 02 Suppl A.)

$$\rho\left[\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right] = -\frac{\partial P}{\partial x} + \eta\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x, \qquad (2.63)$$

$$\rho\left[\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right] = -\frac{\partial P}{\partial y} + \eta\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y, \qquad (2.64)$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial P}{\partial z} + \eta\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z. \qquad (2.65)$$

Use

$$v^* = \frac{v_y}{V_\infty}, \quad y^* = \frac{y}{L}, \quad \mathrm{Re}_L = \frac{V_\infty L}{\nu}, \quad \mathrm{Fr}_L = \frac{V_\infty^2}{g_y L}$$

Ref: p. 64 of the textbook

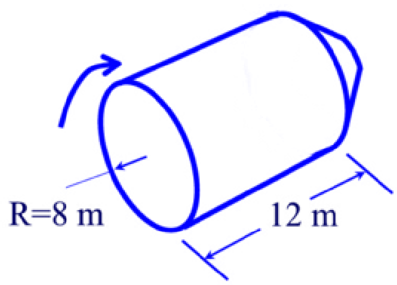
Just do the second term in each bracket since they will be the same factor for every term in the same bracket.

$$\rho\left[..+Vv_x^*\frac{V\partial v_x^*}{D\partial x^*}+...\right]=-\frac{\eta V}{D^2}\frac{dP^*}{dx^*}+\eta\left[..+\frac{V\partial^2 v_x^*}{D^2\partial y^{*2}}+..\right]+\rho g_x$$

$$\frac{V^2}{D}\rho\left[..+v_x^*\frac{\partial v_x^*}{\partial x^*}+...\right]=-\frac{\eta V}{D^2}\frac{dP^*}{dx^*}+\eta\frac{V}{D^2}\left[..+\frac{V\partial^2 v_x^*}{X\partial y^{*2}}+..\right]+\rho g_x$$

$$\left[..+v_x^*\frac{\partial v_x^*}{\partial x^*}+...\right]=-\frac{1}{\mathrm{Re}}\frac{dP^*}{dx^*}+\frac{1}{\mathrm{Re}}\left[..+\frac{\partial^2 v_x^*}{\partial y^{*2}}+..\right]+\frac{1}{Fr_x}=0$$

3. One of SoDak Metal Corporation's many processes involves the recovery of precious metals from a viscous Newtonian slag. The molten slag, contained in the vessel shown above, is first mixed with a mixture of chemicals, which prepare the slag for further processing, and melted. The melted addition has a viscosity three times greater than the slag to be conditioned. Mixing is achieved by rotating the vessel about its axis at 20 RPM ($\omega = 2\pi/3$ rad/s). Since precious metal recovery in the overall process has been poor, the completeness of mixing in this step of the process is now being investigated.



You have decided to model the system by using water in a transparent vessel. By adding a quantity of viscous syrup that contains an acidic component and an acid/base indicator (blue above pH 7.23) to a strongly basic solution in the vessel, you will determine when mixing is complete by the disappearance of the blue indicator. When thoroughly mixed the final pH is just enough below 7.23 to eliminate all the blueness in the solution. Tip: establish a Velocity with units of distance/time such as (RPM*D).

a) Determine the RPM, length, and diameter of the vessel used for modeling the mixing process. Clearly state your choice of variables.

Given:	η_{SLAG} = 333 cp	$\rho_{SLAG} = 3.00\ g/cm^3$
	η_{H_2O} = 1.00 cp	$\rho_{H_2O} = 1.00\ g/cm^3$

b) If the complete mixing takes 1.00 minute in the bench scale mixer, how much time would be required to fully mix in the process mixer?

$$V_i = k\varpi_i D_i$$

$$Fr_S = Fr_L$$

$$\frac{D_L \varpi_L^2}{D_S \varpi_S^2} = 1$$

$$\frac{\varpi_L}{\varpi_S} = \sqrt{\frac{D_S}{D_L}}$$

$$\mathrm{Re}_S = \mathrm{Re}_L$$

$$\frac{D_S^2 k \varpi_S \rho_S}{\eta_S} = \frac{D_L^2 k \varpi_L \rho_L}{\eta_L}$$

$$\frac{D_S^2 \varpi_S \rho_S \eta_L}{D_L^2 \varpi_L \rho_L \eta_S} = 1$$

$$\left(\frac{D_S}{D_L}\right)^{3/2} = \frac{\rho_S \eta_L}{\rho_S \eta_L} = \frac{1}{111}$$

c)

$$D_S = 16 * m * \left(\frac{1}{111}\right)^{2/3} = 0.692 m(diameter)$$

$$\varpi_S = \varpi_L \sqrt{\frac{D_L}{D_S}} = 20 RPM * \sqrt{\frac{16}{0.692}} = 96.1 RPM$$

$$t_L = t_S \frac{\varpi_S}{\varpi_L} = 1 \min \frac{96.1}{20} = 4.81 \min$$