SOUTH DAKOTA SCHOOL OF MINES AND TECHNOLOGY

Department of Materials and Metallurgical Engineering

Met 422 **Buckingham Pi Theory** Oct 22, 2020

The Buckingham Pi Theory was presented in modern form by American physicist Edgar Buckingham born in Philadelphia in 1867[1]. The theory was a formalization of Sir Raleigh's first proven by French mathematician Joseph Bertrand in 1878 but who only considered special cases applied to electrodynamics. A. Vaschy in 1892 and both A. Federman and D. Riabouchinsky in 1911 independently extended previous descriptions and formulations of the method as did Buckingham in 1914 who added the use of Pi groups (*Trans* ASME v 35 (262) 1915).

Formal Statement

For a physical meaningful system

$f(q_1, q_2, q_3, \ldots\ q_n) = 0$

where q_i are the independent physical variables in u independent units may be written

$F(\pi_1, \pi_2, \ldots\ \pi_{n-u}) = 0$

where each π value is made dimensionless by rational ai values

$\pi_i = q_1^{a1} q_2^{a2} q_3^{a3} \ldots\ q_n^{an}$.

Application Example and Rules

For a physical meaningful function

$q_0 = f(q_1, q_2, q_3, \ldots\ q_n)$

with u fundamental, and therefore independent, units, select u number of so-called primary q's such that

1) All units are represented and
2) The units of any primary q cannot be obtained from the other primary q's.

Then use the primary q's, now denoted as r_i's, to eliminate the units in q_0 and the (n-u) non-primary q's via soling for ai's in

$\pi_I = q_i/(r_1^{a1} r_2^{a2} r_3^{a3} \ldots r_U^{au})$

Then

$\pi_0 = f(\pi_1, \pi_2, \ldots\ \pi_{n-u})$

Example

Given $h = f(\rho, Cp, k, \eta, D, V)$

Choose as primary q's ρ, Cp, D, V

Then

$$\pi_0 = \frac{h}{\rho^{a0} Cp^{b0} D^{c0} V^{d0}}$$

$$\pi_1 = \frac{\eta}{\rho^{a1} Cp^{b1} D^{c1} V^{d1}}$$

$$\pi_2 = \frac{k}{\rho^{a2} Cp^{b2} D^{c2} V^{d2}}$$

The fundamental units for each q are as follows:

1. (https://en.wikipedia.org/wiki/Edgar_Buckingham

	ρ^a	Cp^b	D^c	V^d	h	η	k
s	0	-2b	0	-d	-3	-1	-3
m	-3a	2b	c	d	0	-1	1
kg	a	0	0	0	1	1	1
K	0	-b	0	0	-1	0	-1

Solving for a, b, c, and d for each shaded RHS provides the desired Pi groups.

$$\pi_0 = \frac{h}{\rho CpV} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}}$$

$$\pi_1 = \frac{\eta}{\rho DV} = \frac{1}{\mathrm{Re}}$$

$$\pi_2 = \frac{k}{\rho CpDV} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}$$

which may be written

$$Nu = f(\mathrm{Re}, \mathrm{Pr})$$