

Packed Beds

Wednesday, September 16, 2020 12:51 PM

w = VOID FRACTION

$$S_o = \frac{\text{Part. Area}}{\text{Part Vol}}$$

$$S = \frac{\text{Bed Surf Area}}{\text{Bed Vol}} = S_o(1-w)$$

HCP = 0.28, typical 0.4

Sphere $\frac{4\pi R^2}{4/3\pi R^3} = \frac{3}{R} = \frac{6}{D_p}$

Cube $\frac{6D_p^2}{D_p^3} = \frac{6}{D_p}$

R_h = Hydraulic Radius = $\frac{\text{Area thru which flow occurs}}{\text{Wetted Perimeter}} = \frac{A_f}{P_w}$

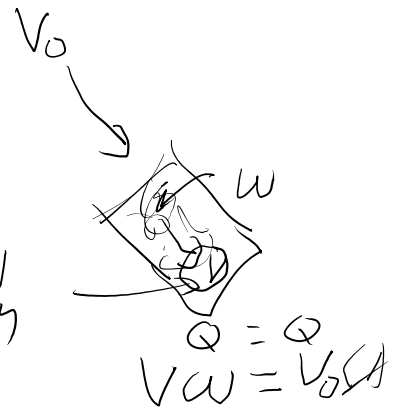
$$= \frac{A_f \times L}{P_w \times L} = \frac{\pi R^2 L w}{\pi R^2 L S}$$

$$R_h = \frac{w}{S_o(1-w)}$$

V_o = superficial velocity

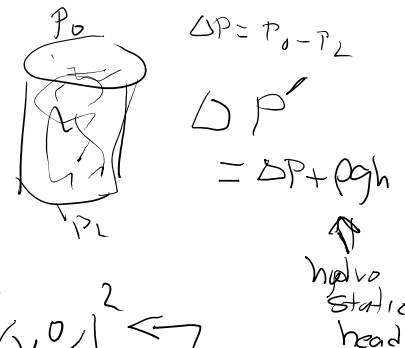
$$V_o = V w$$

interstitial velocity



$$Re = \frac{V D_p}{\eta} = \frac{\left(\frac{V_o}{w}\right) \rho (w)}{\eta (S_o(1-w))}$$

$$Re = \frac{V_o \rho}{\eta S_o(1-w)}$$



$$F_c = \frac{F_k}{A K} = \frac{\pi R^2 \Delta P w}{S_o(1-w) \rho \times \left(\frac{V_o}{w}\right)^2}$$

$$F_c = \frac{\Delta P' w^3}{150 \eta (1-w) v^2}$$

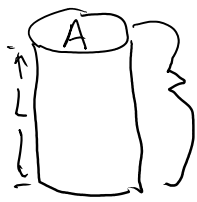
EQ. (3.44)

$A = \text{wetted Area}$
 $A = \pi R^2 L S$
 $A = \pi R^2 L S_o(1-w)$
 $K = \frac{\rho \gamma}{\eta v^2}$

$$\boxed{t_c = \frac{\Delta P w}{L S_o (1-w) V_o^2 \rho}}$$

(3.44) $\boxed{K = \frac{\rho r}{8 \eta}}$
 "1/2" Not Used $\boxed{K = \rho r^2 / 8 \eta^2}$

OHM'S LAW APPROACH



$$Q = K_D \frac{\Delta P'}{L} A$$

$$Q = \frac{\rho}{\eta} \frac{\Delta P'}{L} A$$

OHM'S LAW

$$I = \frac{\Delta V}{R}$$

$$Q = \frac{\Delta P}{\left(\frac{L}{A K_D}\right)}$$

$$V_o = \frac{\rho}{\eta} \frac{\Delta P}{L}$$

Permeability

$$\boxed{1 \text{ Darcy} = 10^{-12} \text{ m}^2}$$

TUBE BUNDLE THEORY

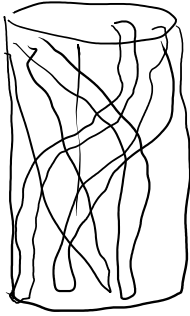
Based on Hagen-Poiseuille Eq (2.33)

$$\underbrace{\bar{V}}_{\text{tube bundle}} = \underbrace{\left[\frac{P_o - P_L}{L} + \rho g \right]}_{\text{}} \frac{R^2}{8 \eta}$$

use R_h

↑ include in K_1

tube bundle



include in K_1

$$V = K_1 \left(\frac{\Delta P'}{L} \right) \frac{\left(\frac{w}{S_0(1-w)} \right)^2}{\eta}$$

(experimentally measured $K_1 = \frac{1}{4.2}$)

$$\frac{V_0}{w} = K_1 \frac{\Delta P'}{L} \frac{w^3}{S_0^2 (1-w)^2 \eta}$$

BLAKE - KOZENY EQUATION

$$V_0 = \frac{1}{4.2} \frac{\Delta P'}{L \eta S_0^2} \frac{w^3}{(1-w)} \quad \text{EQ. (3.41)}$$

ERGUN EQUATION: CONSIDERING TURBULENT FLOW

$$\frac{\Delta P'}{L} = \frac{4.2 \eta S_0^2 (1-w)^2 V_0}{w^3}$$

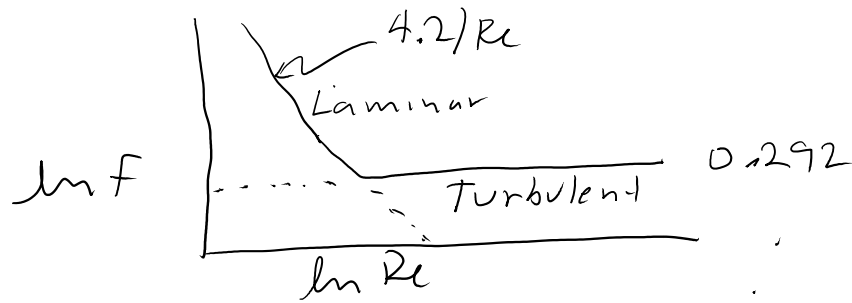
← left out in 09-16 class video
from Blake-Kozeny Eq

$$\Delta P \pi R^2 w = F_R = f_c A K = f_c \pi R^2 L S_0 (1-w) \rho \frac{V_0^2}{w^2}$$

$$f_c = \frac{\Delta P'}{L} \frac{w^3}{S_0 \rho (1-w) V_0^2} \quad \text{Eq (3.41)}$$

Experimental Results

Experimental Results



$$f_c = \underbrace{\frac{4.2}{Re}}_{\text{laminar term}} + \underbrace{0.292}_{\text{turbulent term}}$$

$$\frac{\Delta P'}{L} = \underbrace{\frac{4.2 \eta V_0 S_0^2 (1-w)^2}{w^3}}_{\text{Lam}} + \underbrace{\frac{0.292 \rho V_0^2 S_0 (1-w)}{w^3}}_{\text{Turb}}$$

Eq (3.46)

↖ Ergun Eq Precursor

Post Lecture Note

$$\frac{\Delta P' \pi R^2 w}{L} = \frac{F_K}{L} = \frac{f_c A K}{L}$$

$$\frac{\Delta P'}{L} = \frac{f_c A K}{\pi R^2 L w} = \left(\frac{4.2}{Re} + 0.292 \right) \frac{\pi R^2 L S_0 (1-w) \rho V_0^2}{\pi R^2 L w^3}$$

$$\frac{\Delta P'}{L} = \left(\frac{4.2}{Re} + 0.292 \right) \frac{S_0 (1-w) \rho V_0^2}{w^3}$$

↙ Laminar term ↘ Turbulent term

EQ (3.46)

$$\frac{\Delta P'}{L} = \frac{4.2 \eta V_0 S_0^2 (1-w)^2}{w^3} + \frac{0.292 \rho V_0^2 S_0 (1-w)}{w^3}$$

$$\text{Eq (3.46)} \quad \left(\frac{\Delta P}{L} = \frac{4.27 \eta V_0 \lambda^2 (1-w)}{w^3} + \frac{0.474 \rho V_0^2 \lambda^2 (1-w)}{w^3} \right)$$

$$\text{If } S_0 = \frac{6}{D_p}$$

λ = shape factor to adj from spheres to non-spheres, See Table 3.2.

$$S_0 = \frac{6\lambda}{D_p}$$

Engun Eq

Eq (3.49)

when $\lambda=1$
i.e. for spherical packing

$$\frac{\Delta P}{L} = \frac{150^* \eta V_0 \lambda^2 (1-w)^2}{D_p^2 w^3} + \frac{1.75 \rho V_0^2 \lambda^2 (1-w)}{D_p w^3}$$

* Really 151.2. but rounded to 150, which is within exp uncertainty.
 $4.2 \times 6^2 = 151.2$

If one has a bed of different size particles, see Eqs (3.53)-(3.55).